CSI 35, Homework 4 on section 5.1

Due by Wed, Mar 5.

Please use lots of space and explain your answers, showing clearly any work you had to do. Each question is worth 5 points.

Section 5.1 Induction

(1) A *proposition* is a statement giving a fact that is either true or false (and not both).

- **(a)** Why is the statement "Turn right at the next corner" not a proposition?
- **(b)** Is the proposition "$x^2 - x + 3 = 1$ when $x = 2$" true or false?
- **(c)** Is the proposition "$(-1)^{1000019} = -1$" true or false?

(2) Suppose, in a large hot star, when 4 hydrodgen atoms combine into one helium atom then the energy produced always causes 4 more hydrogen atoms to combine into a helium atom. If this star reaches a temperature to cause the first 4 hydrodgen atoms to combine into a helium atom, what happens after that?

(3) Prove the formula

$$\underbrace{(-7) + (-7) + \cdots + (-7)}_{n \text{ of these}} = -7n$$

for $n \geqslant 1$ by induction using these steps:

- **(a)** Identify the proposition we want to prove and write "$P(n)$ says ....".
- **(b)** Check the *basis step*.
- **(c)** Complete the *inductive step* as follows: write down what $P(k)$ says, assume it's true, and then use this to prove that $P(k+1)$ must also be true.
- **(d)** Write your conclusion: "So by mathematical induction $P(n)$ is true for all $n \geqslant 1$".

(4) Use the same four step as the last question to prove by induction that

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

is true for all $n \geqslant 1$.

(5) Consider the sum

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n(n+1)}.$$

(a) When $n=1$ the sum is just

$$\frac{1}{1\cdot 2}=\frac{1}{2}.$$

Compute the fraction the sum equal to when $n=2$:

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}=$$

(b) Compute the fraction the sum equal to when $n=3$:

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}=$$

(c) Use parts (a) and (b) to conjecture (guess) what the formula for the sum should be for any n.

(6) Use induction to prove your conjectured formula from the last question.

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes or section in the textbook.
- Ask me about it after class.
- Come to my office hours: Mon 2:00 - 3:00, Wed 2:00 - 3:00 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.