CSI 35, Homework 3 on sections 9.5, 9.6

Due by Wed, Feb 26.

Please use lots of space and explain your answers, showing clearly any work you had to do. Each question is worth 5 points.

Section 9.5 Equivalence relations

(1) Equivalence relations are reflexive, symmetric and transitive. Say which of these relations, on the set of all people, are equivalence relations and explain why:

(a) $\{(a, b) \mid a$ and b are the same age$\}$

(b) $\{(a, b) \mid a$ and b speak a common language$\}$

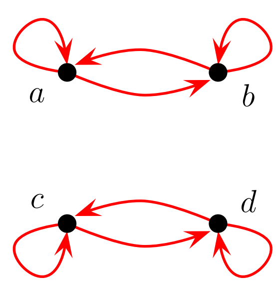
(c) $\{(a, b) \mid a$ and b have met$\}$

(d) $\{(a, b) \mid a$ and b have the same parents$\}$

(e) $\{(a, b) \mid a$ and b were both born in April$\}$

(2) Give an equivalence relation on the set of all buildings in New York City. So write "building A is related to building B if ....". Explain how your relation partitions the NY buildings into different classes.

(3) Decide if the relation on $\{a, b, c, d\}$ given by this digraph is an equivalence relation. If it is, give the distinct equivalence classes and show the partition of $\{a, b, c, d\}$ they make.



(4) Let $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let R be the equivalence relation on A given by

$$R = \{(a, b) \mid a \equiv b \bmod 3\}.$$

Find the distinct equivalence classes of R and show the partition of A they make.

(5) An equivalence relation S on the set $\{a, b, c, d, e, f, g\}$ produces the partition

$$\{a, b\},\ \{c, d\},\ \{e, f, g\}.$$

List the ordered pairs in S.

Section 9.6 Partial orders

(6) Partial orders are reflexive, antisymmetric and transitive. Say which of these relations on the set $\{0, 1, 2, 3\}$ are partial orders and explain why:

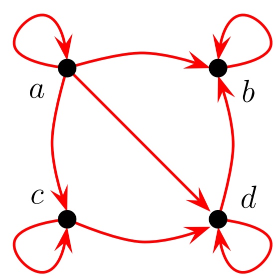
(a) $\{(0, 0), (1, 1), (2, 2), (3, 3)\}$

(b) $\{(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 3)\}$

(c) $\{(0, 0), (1, 1), (1, 2), (2, 2), (3, 1), (3, 3)\}$

(7) Give two partial orders on the set of all buildings in New York City and explain why they are partial orders. Write "building A is related to building B if ...." when giving your relations.

(8) Decide if the relation on $\{a, b, c, d\}$ given by this digraph is a partial order. Explain.



(9) Draw the Hasse diagram for divisibility on each of these sets. (For example, 5 divides 15 since it goes in 3 times with no remainder, but 5 does not divide 11 since it goes in 2 times with remainder 1.)

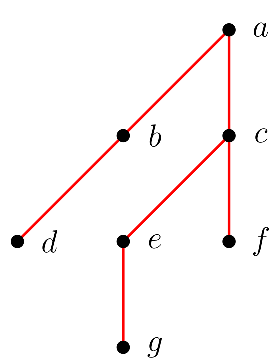
(a) $\{1, 2, 3, 4, 5, 6, 8\}$

(b) $\{3, 5, 7, 11, 13, 16\}$

(c) $\{2, 3, 5, 10, 11, 15, 25\}$

(d) $\{1, 3, 9, 27, 81\}$

(10) For the partial order represented by this Hasse diagram find **(a)** all maximal elements, **(b)** all minimal elements, **(c)** any greatest element, **(d)** any least element.



If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes or section in the textbook.
- Ask me about it after class.
- Come to my office hours: Mon 2:00 - 3:00, Wed 2:00 - 3:00 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.