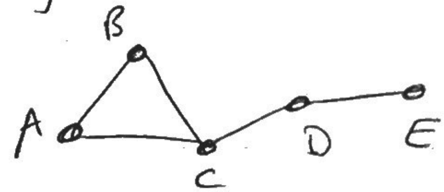
10.1 Graphs and Graph Models.

Suppose A, B, C, D, E represent five cities. We could represent the road network connecting these as



Graph 1

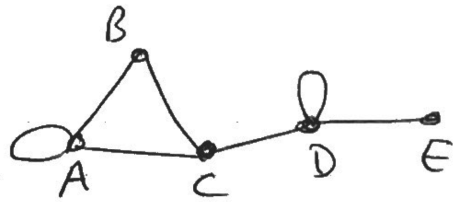
It shows for example that you can drive between A and B, but if you want to drive from A to D you must go through C.

This is an example of a graph with set of vertices (dots) $V = \{A, B, C, D, E\}$ and set of edges (lines)

$$E = \{(A,B), (B,C), (A,C), (C,D), (D,E)\}.$$

So an edge is a pair of vertices, and here the order doesn't matter.

Suppose there was also a scenic costal road that starts and ends at A, and same for D. We could represent this as



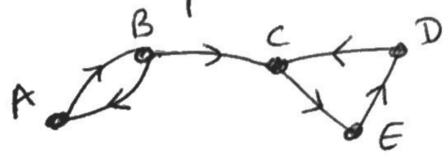
Graph 2

by adding two loops to the picture and (A,A) and (D,D) to the set of edges E.

There could also be more than one road between cities

Graph ③

and if all the roads are one-way we might get a picture



Graph ④

A graph like this where every edge has a direction is called a <u>directed graph</u> (or digraph for short).

Its edges are now ordered pairs and Graph ④ has edge set

$$E = \{(A,B), (B,A), (B,C), (C,E), (E,D), (D,C)\}$$

As we saw in Chapter 9, directed graphs represent <u>relations</u>.

If every edge does not have a direction then the graph is called <u>undirected</u>.

Types of undirected graphs

- A graph like Graph ① with no loops or multiple edges is called <u>simple</u>.
- A graph like Graph ③ with multiple edges but no loops is called a <u>multigraph</u>.

- A graph like Graph 2 with loops and possibly multiple edges is a <u>pseudograph</u>.

	Multiple edges	Loops
Simple graph	No	No
Multigraph	Yes	No
Pseudograph	Yes	Yes

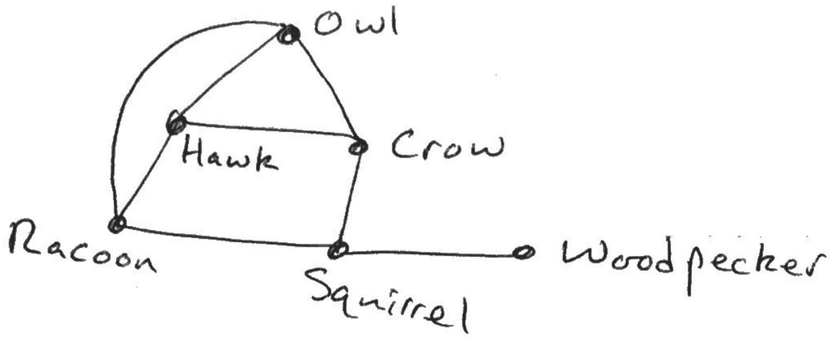
So in general a graph G is a set of vertices V and a set of edges E where the edges are really pairs of vertices.
We write $G = (V, E)$.

Graphs are useful for modeling many things such as roads, communication networks, molecules, airline routes etc. Depending on the example, a simple graph, multigraph, pseudograph or a directed graph might be most appropriate for the model.

<u>Example</u> from ecology

A <u>niche overlap graph</u> can be used to model competition between species in an environment. So the vertices represent species and there is an edge between two species if they compete directly for food or space.

We do not need loops or multiple edges so this is a simple graph



See more examples of graph models in section 10.1 pages 644 – 649.

10.2 Graph Terminology

Graphs have their own language. We must learn it!

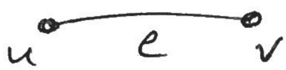
We've already seen

- graph $G = (V, E)$ — V: vertex set, E: edge set
- simple graph
- multigraph
- pseudograph

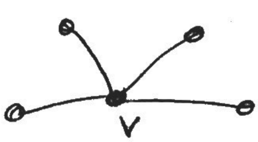
Two vertices in a graph are adjacent (or neighbors) if they are connected by an edge. So

here u and v are adjacent and (u,v) (or (v,u)) is an edge in the graph.

We could label the edge as e



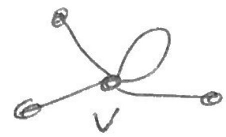
and say that the vertices u and v are the endpoints of e.



$\deg(v) = 4$

The number of edges that have vertex v as an endpoint is called the degree of v with notation $\deg(v)$.

Loops contribute 2 to the degree:

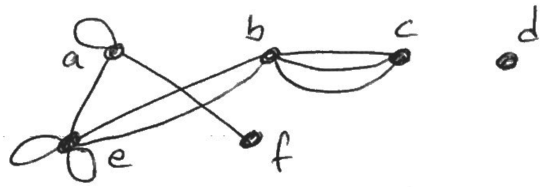


$\deg(v) = 5$

A vertex with degree 0 is called <u>isolated</u>.
A vertex with degree 1 is called <u>pendant</u>.

<u>Example</u>

For this pseudograph $G = (V, E)$



find $|V|$ the number of vertices, $|E|$ the number of edges, the degree of each vertex, the total degree, and identify any pendant or isolated vertices.

<u>Solution</u>

$|V| = 6$
$|E| = 10$

$\deg(a) = 4$
$\deg(b) = 5$
$\deg(c) = 3$
$\deg(d) = 0$ ← so d is isolated
$\deg(e) = 7$
$\deg(f) = 1$ ← f is pendant

total degree 20