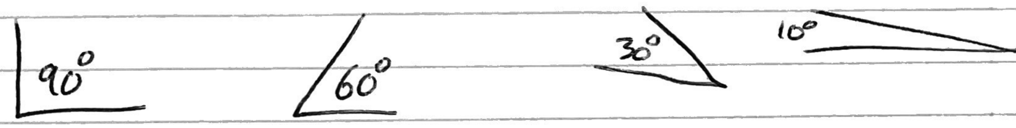
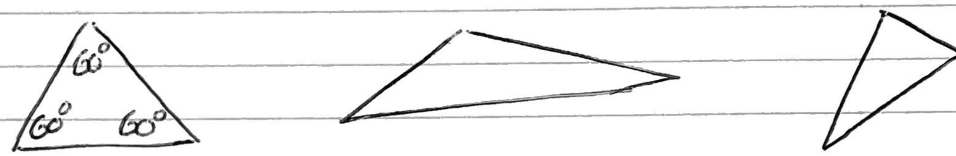
# Right triangle trigonometry Part I

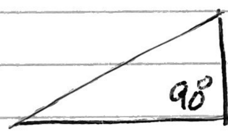
Two lines form an angle, measured in degrees:



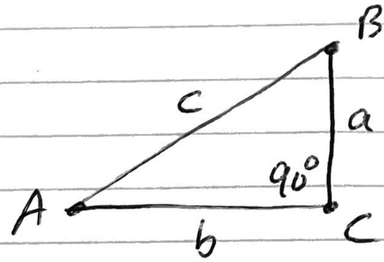

The angles inside any triangle add up to 180°




If a triangle contains a 90° angle (a right angle) then it is called a right triangle.

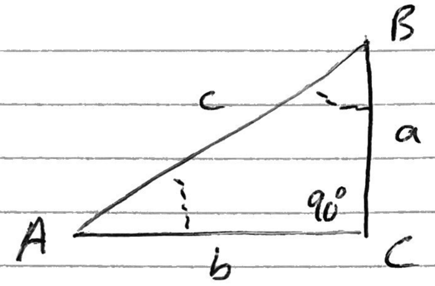



These are the triangles used in basic trigonometry. The standard set up looks like this




Use A, B, C for the points of the triangle and a, b, c for the sides.

We also use A, B, C for the angles there. Side a is opposite angle A and same for b, B and c, C where angle C is always 90°.




Side c opposite the 90° angle is called the hypotenuse. The two shorter sides a, b are the legs.

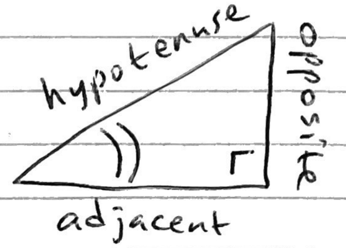
The sum of the angles is 180°

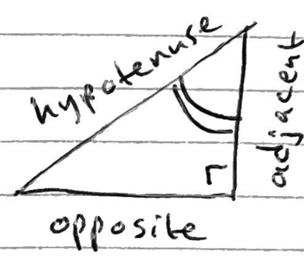
$$A + B + C = 180^\circ$$

and $C = 90^\circ$ so $A + B = 90^\circ$.

- The Trigonometric Ratios

The next step is to link the angles in a right triangle to its sides. The angles at A and B must be less than 90° and we look at these. They have the hypotenuse on one side, and the other side making the angle is called adjacent. The third side is called opposite.



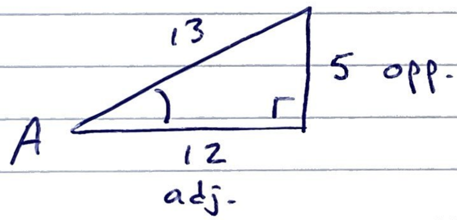
So sides are adjacent or opposite depending on which angle you are using. The hypotenuse is fixed.

The three main trigonometric ratios of an angle are

$$\text{sine} = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \text{cosine} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{and} \qquad \text{tangent} = \frac{\text{opposite}}{\text{adjacent}}.$$

For example

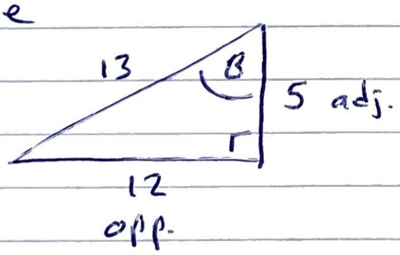



$$\sin A = \frac{5}{13}$$

$$\cos A = \frac{12}{13}$$

$$\tan A = \frac{5}{12}$$

while




$$\sin B = \frac{12}{13}$$

$$\cos B = \frac{5}{13}$$

$$\tan B = \frac{12}{5}$$

Notice the opposite and adjacent switched for angles A and B.

Use "Soh Cah Toa" or "Some old horses can always hear their owners approach" to help remember these ratios.

There are 3 more trig. ratios to remember, reciprocals of cos, sin, tan:

$$\text{secant} = \frac{1}{\text{cosine}} = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\text{cosecant} = \frac{1}{\text{sine}} = \frac{\text{hypotenuse}}{\text{opposite}}$$

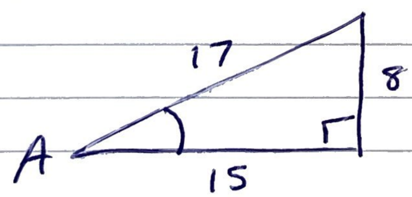
$$\text{cotangent} = \frac{1}{\text{tangent}} = \frac{\text{adjacent}}{\text{opposite}}$$

ie.

$$\sec A = \frac{1}{\cos A} \qquad \csc A = \frac{1}{\sin A} \qquad \cot A = \frac{1}{\tan A}$$

s and c switched, to help remember

Example ② Give all six trig. ratios for the angle shown




Answer:

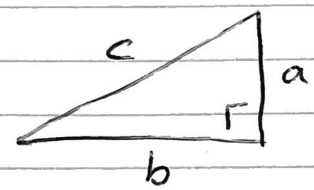
$$\sin A = \frac{8}{17} \quad \cos A = \frac{15}{17} \quad \tan A = \frac{8}{15}$$

$$\sec A = \frac{17}{15} \quad \csc A = \frac{17}{8} \quad \cot A = \frac{15}{8}.$$

The trig. ratios are related in different ways. An important relation is

$$\tan A = \frac{\sin A}{\cos A} \quad \text{for any angle } A.$$

- The Pythagorean Theorem.

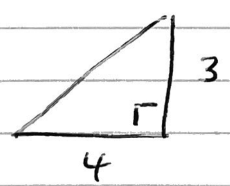



This theorem gives the relationship between the side lengths in a right triangle:

$$\underbrace{a^2 + b^2}_{\text{legs}} = \underbrace{c^2}_{\text{hypotenuse}}$$

So if we know two sides then we can use this relationship to find the third.

Example 3 Find the missing side:




Solution: we see the hypotenuse is missing so

$$3^2 + 4^2 = c^2$$

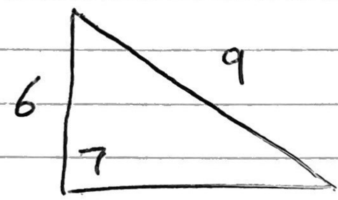
$$9 + 16 = c^2$$

$$25 = c^2 \quad \text{and} \quad c = \pm\sqrt{25}$$

by the square root property. Side lengths must be positive so $c = 5$.

Answer: the missing side has length 5.

Example ④ Find the missing side:




Solution: Now one of the legs is missing. Let $x$ be the missing number.

$$6^2 + x^2 = 9^2$$

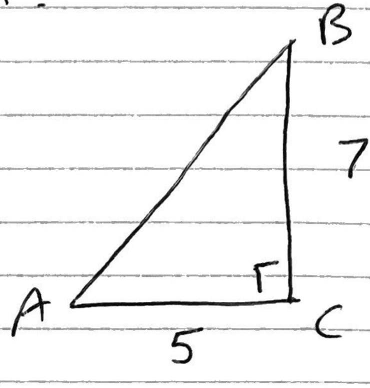
$$36 + x^2 = 81, \qquad x^2 = 45$$

then $x = \sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}$ simplified

Answer: the missing side has length $3\sqrt{5}$

Keep your answer as an exact radical, no need to give a decimal unless asked.

Example ⑤ The diagram shows the right triangle ABC with $C = 90°$. Find $\tan B$ and $\sin A$.



Solution: For angle B we already know the opposite and adjacent so

$$\tan B = \frac{5}{7}.$$

For $\sin A$ we need the hypotenuse. By the Pythagorean Theorem

$$5^2 + 7^2 = c^2 \qquad c = \text{hyp.}$$

$$25 + 49 = 74 = c^2 \qquad c = \sqrt{74}$$

Then $\sin A = \dfrac{\text{opp.}}{\text{hyp}} = \dfrac{7}{\sqrt{74}}$

To simplify this

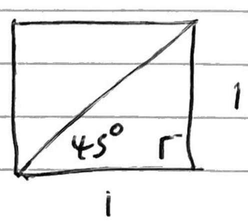
$$\frac{7}{\sqrt{74}} = \frac{7\sqrt{74}}{\sqrt{74}\sqrt{74}} = \frac{7\sqrt{74}}{74}$$

(remember, there should be no radicals in the denominator.)

Answer: $\tan B = \frac{5}{7}$, $\sin A = \frac{7\sqrt{74}}{74}$.

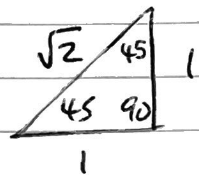
- Special angles

Start with a square of side length 1 and draw a diagonal line:




By the Pythagorean Theorem this diagonal has length $\sqrt{2}$

This makes the 45-45-90 triangle

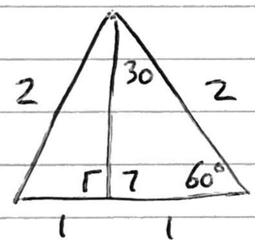



So that $\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ simplified

$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

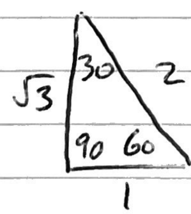
$\tan 45° = \frac{1}{1} = 1$

Starting with an equilateral triangle and drawing a vertical:




By the Pythagorean Theorem again, the vertical has length $\sqrt{3}$

This makes the 30-60-90 triangle




use to find any trig. ratio of 30° or 60°

Can make a table

| $\theta$ | $\sin\theta$ | $\cos\theta$ | $\tan\theta$ |
|---|---|---|---|
| 30° | $1/2$ | $\sqrt{3}/2$ | $\sqrt{3}/3$ |
| 45° | $\sqrt{2}/2$ | $\sqrt{2}/2$ | 1 |
| 60° | $\sqrt{3}/2$ | $1/2$ | $\sqrt{3}$ |

Greek letter $\theta$, "theta" for angles.

So the special angles of $30^\circ$, $45^\circ$ and $60^\circ$ have exact values for their trig. ratios.

For example

$$\tan 30^\circ + \tan 60^\circ = \frac{\sqrt{3}}{3} + \sqrt{3} = \frac{4\sqrt{3}}{3}.$$

Other angles have trig. ratios that may be found on a calculator as decimals that are not exact:

$$\cos 26^\circ = 0.8988\ldots$$

$$\sin 81^\circ = 0.9877\ldots$$

Make sure your calculator is in degrees mode.