3.3 Basic rules of probability

Remember our set-up: for any experiment there is a set S of all possible outcomes, called the sample space.

Events E are subsets of S and their probability is

$$\boxed{P(E) = \frac{n(E)}{n(S)}}$$

← numbers of elements in each

probability formula

For example 1 our experiment of rolling a die. Then $S = \{1, 2, 3, 4, 5, 6\}$ possible outcomes

If E is the event of rolling 4 or lower then $E = \{1, 2, 3, 4\}$ and

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{6} = \frac{\cancel{2}\cdot 2}{\cancel{2}\cdot 3} = \frac{2}{3} = 0.66..$$

probability of rolling 4 or lower is 0.66

Also, every event E has a complementary event E' which means E not happening.

$P(E')$ here means the probability of not rolling 4 or lower.

Since $E' = \{5, 6\}$, we see $P(E') = \frac{n(E')}{n(S)}$

$$= \frac{2}{6} = \frac{1}{3} = 0.33..$$

In general, we always have

$$\boxed{P(E') = 1 - P(E)}$$

complementary probability formula.

Ex.(2) Pick a random jelly bean from a jar containing
6 yellow, 9 red and 15 green beans.

a) What is the probability of picking a red one?
b) What is the probability of not picking a red?

Solution. S = all 30 beans

take E = all 9 red beans

then $P(E) = \frac{n(E)}{n(S)} = \frac{9}{30} = \frac{\cancel{3}\cdot 3}{\cancel{3}\cdot 10} = 0.3$,

also $P(E') = 1 - P(E)$
$= 1 - 0.3 = 0.7$

Answer: probability of red is 0.3 and probability of not picking red is 0.7.

Ex.(3) Roll a pair of dice. Find the probability the sum of two numbers is 7.

Solution. To understand the sample space for this experiment we can list all possible outcomes.

they are pairs of numbers - from the first die and the second:

S

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

E

So $n(S) = 36$ and if E is the event of getting sum 7 we see $n(E) = 6$

$$P(E) = \frac{6}{36} = \frac{1}{6} = 0.166 = 16.6\%$$

So 16% chance of getting sum 7.

For this same experiment we can also ask the probability of rolling a double - meaning both die the same number, like (4,4). Let F be this event

then $P(F) = \frac{6}{36} = \frac{1}{6} = 16.6\%$ also.

<u>Combining events</u>

$P(E \wedge F)$ means the probability of both events E, F happening together

set intersection

$$= \frac{n(E \wedge F)}{n(S)}$$

In our last example $E \wedge F = \{\}$ empty

$$\text{so} \quad P(E \wedge F) = \frac{0}{36} = 0$$

This means rolling a sum 7 and a double has probability 0 (it's impossible).

If $P(E \wedge F) = 0$ we say E and F are mutually exclusive.

(They cannot happen together.)

Ex (4) Pick a random card from the pack. Let E be the event it is a diamond. Let F be the event it's a ten.

Find a) $P(E)$ b) $P(F)$ c) $P(E \wedge F)$

Solution. Check that

$$P(E) = \frac{n(E)}{n(S)} = \frac{13}{52} = \frac{1}{4} = 0.25$$

$$P(F) = \frac{n(F)}{n(S)} = \frac{4}{52} = \frac{1}{13} = 0.077$$

$$P(E \wedge F) = \frac{n(E \wedge F)}{n(S)} = \frac{1}{52} = 0.019$$

here $E \wedge F = \{\text{ten of diamonds}\}$.

Another way to combine events:

$P(E \cup F)$ means the probability of E or F happening, or both.

↑ set union

$$P(E \cup F) = \frac{n(E \cup F)}{n(S)} = \frac{n(E) + n(F) - n(E \cap F)}{n(S)}$$

so we get a formula

$$\boxed{P(E \cup F) = P(E) + P(F) - P(E \cap F)}$$

In words, the probability of E or F happening is $P(E) + P(E)$ minus the probability of E and F both happening.

So from our last example, the probability of picking a diamond or a ten is

$$P(E \cup F) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}.$$