Mth 31, Homework 9 on sections 4.3, 4.4

Due by Wed, Nov 12.

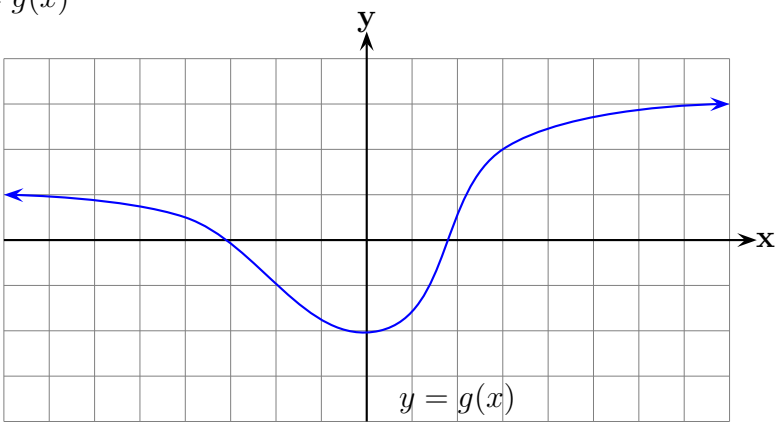
Write all your working out and answers clearly and neatly, using lots of space. Each question is worth 3 points.

Section 4.3 How derivatives affect graph shape

(1) Let $f(x) = -x^2 + 2x + 15$.

- **(a)** Use the first derivative to see the xs where f is increasing and decreasing. Give your answer in interval notation.
- **(b)** Find all the local maximums and minimums: identify which is which and give their coordinates.

(2) For this graph $y = g(x)$



- **(a)** Give the intervals where g is increasing and decreasing.
- **(b)** Identify and locate all local maximums and minimums.
- **(c)** Give the intervals where g is concave up and down.
- **(d)** Locate all inflection points.

(3) Let $f(x) = x^3 - 12x^2 - x$.

- **(a)** Use the second derivative to see the xs where f is concave up and down. Give your answers in interval notation.
- **(b)** Locate all inflection points.

(4) Draw the graph of $\sin x$ on the interval $[0, 2\pi]$. From your graph, find the intervals where it is increasing/decreasing and where it is concave up/down.

(5) Let $G(x) = \ln(x^2 + 4)$.

(a) Give the intervals where G is increasing and decreasing.

(b) Identify and locate all local maximums and minimums.

(c) Give the intervals where G is concave up and down.

(d) Locate all inflection points.

(e) Sketch the graph of $G(x)$ carefully using the information found in parts (a) – (d).

(6) The latest unemployment numbers show that the number of unemployed people is increasing, but at a decreasing rate. Say what this statement means in terms of the derivatives of the function $U(t)$, where $U(t)$ is the number unemployed at time t.

Section 4.4 Indeterminate forms and l'Hospital's rule

(7) Suppose $f(0) = 0$ and $g(0) = 0$. Also, at the point $(0, 0)$, $f(x)$ has tangent line $y = 2x$ and $g(x)$ has tangent line $y = 3x$. Use this to find: $\lim_{x\to 0} \dfrac{f(x)}{g(x)}$

(8) Use l'Hospital's rule to calculate: **(a)** $\lim_{x\to 9} \dfrac{\sqrt{x} - 3}{x - 9}$ **(b)** $\lim_{x\to \infty} \dfrac{\sqrt{x} - 3}{x - 9}$

(9) Use l'Hospital's rule if it applies, or another method, to calculate:

(a) $\lim_{x\to 0} \dfrac{\sin(4x)}{\sin(3x)}$ **(b)** $\lim_{x\to 0} \dfrac{\cos(4x)}{\cos(3x)}$

(10) Compute: $\lim_{x\to 0} \dfrac{\cos(x) - 1}{x^2}$

(11) Calculate: $\lim_{\theta\to 0} \dfrac{\tanh(\theta^2)}{\tan(\theta^2)}$

(12) Calculate: **(a)** $\lim_{t\to 0} \dfrac{6^t - e^t}{6t}$ **(b)** $\lim_{x\to \infty} x^2 \cdot 3^{-x}$

(Those ts are both powers on the top of part (a).)

(13) Find: $\lim_{x\to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{e^x - 1} \right)$

(Hint: combine these using a common denominator first. Then apply l'Hospital.)

(14) Find: **(a)** $\lim_{x\to \infty} \dfrac{\ln(x)}{x}$ **(b)** $\lim_{x\to \infty} x^{1/x}$

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.
- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).
- Ask me about it after class.
- Come to my office hours: Mon 11:30 - 12:30, Wed 11:30 - 12:30 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.