Write all your working out and answers clearly and neatly, using lots of space. Each question is worth 3 points.

Section 4.1 Maximum and minimum values

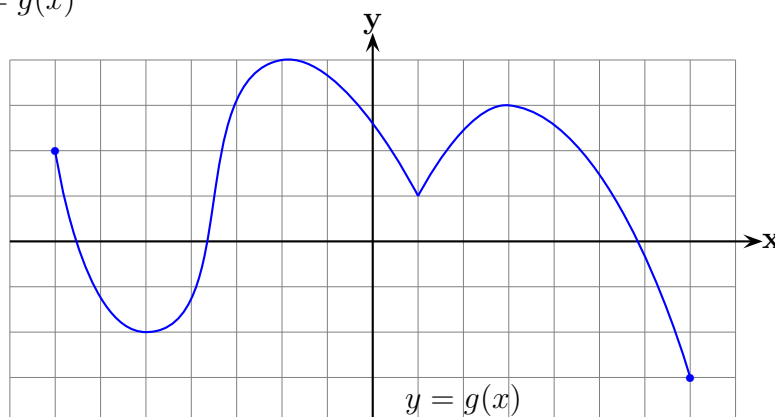**(1)** Let $f(x) = x^3 - 2x - 4$. Find the values of this function

   (a) at $x = 2$

   (b) at $x = -3$

   (c) at $x = 1/2 = 0.5$

(2) Which of the numbers -2, 1.4, -1, 6, 3.001, -0.99 are in the closed interval $[-1, 3]$?

(3) For this graph $y = g(x)$

   (a) Identify all the local maximums and minimums of $g(x)$ and give the coordinates of these points (there are 4 of them). Remember that end-points are never called local maximums or minimums.

   (b) Give the values of g at these local maximums and minimums (these are the ys).

   (c) Identify the absolute maximum and minimum of $g(x)$ and give the coordinates of these points. (Can be end-points.)

   (d) Give the absolute maximum and minimum values of g.

(4) Sketch the graph of $f(x) = x^2 - 1$ for x in the closed interval $[-1, 2]$. Use your graph to find the absolute maximum and minimum values of the function on this interval and identify all local maximums and minimums.

(5) Remenber that a *critical number* for $f(x)$ is a c where $f'(c) = 0$ or $f'(c)$ does not exist.

    (a) Find all the critical numbers of $g(x)$ in question (3). These critical numbers are xs.

    (b) Find the critical numbers of: $F(x) = x^3 - 27x + 9$

    (c) Find the critical numbers of: $G(x) = |3x + 12|$

(6) Use the closed interval method to find the absolute maximum and minimum values of $f(x) = 2x^3 - 3x^2 - 12x + 1$ on $[-2, 3]$.

(7) Use the closed interval method to find the absolute maximum and minimum of

$$h(x) = \frac{x}{x^2 + 1} \quad \text{on} \quad [0, 3]$$

(Hint: when you look at $h'(x)$ remember that the only way for a rational function to be zero is if the numerator (top) is zero.)

Section 4.2 The Mean Value Theorem

(8) Your flight from New York to Los Angeles takes 6 hours to go 2400 miles. What is your average velocity? What does the *Mean Value Theorem* say about your flight?

(9) Let $f(x) = -2x^2 + x + 1$.

    (a) What exactly does the Mean Value Theorem say about $f(x)$ on the interval $[-1, 3]$?

    (b) Find all the numbers c that satisfy the conclusion of the theorem.

(10) Let $g(x) = x^3 - 3x + 2$.

    (a) What exactly does the Mean Value Theorem say about $g(x)$ on the interval $[-2, 2]$?

    (b) Find all the numbers c that satisfy the conclusion of the theorem.

(11) Let $h(x) = 2 + \ln x$.

    (a) What exactly does the Mean Value Theorem say about $h(x)$ on the interval $[1, 10]$?

    (b) Find all the numbers c that satisfy the conclusion of the theorem.

(12) Suppose $f(2) = -3$ and $f'(x) \geqslant 3/2$ for $2 \leqslant x \leqslant 6$. Use the Mean Value Theorem to find the smallest possible value of $f(6)$.

(13) Show that the equation
$$x^3 - 12x + 3 = 0$$
has exactly one solution in the interval $[-1, 1]$ with the following steps.

    (a) Use the Intermediate Value Theorem for $f(x) = x^3 - 12x + 3$ on $[-1, 1]$ to show that there is at least one solution in the interval.

(b) Now suppose there are two solutions a and b in the interval with $a < b$. In other words $f(a) = 0$ and $f(b) = 0$. Apply the Mean Value Theorem to $f(x)$ on $[a, b]$. What does it say?

(c) Explain why what is says cannot be true. This means there cannot be two solutions – there is exactly one (though we don't know what it is).

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.

- Check if you get the right answer for a similar odd-numbered question in the text-book (answers at the back of the book).

- Ask me about it after class.

- Come to my office hours: Mon 11:30 - 12:30, Wed 11:30 - 12:30 in CP 317.

- Go to the Math Tutorial Lab in-person in CP 303 or online.