Write all your working out and answers clearly and neatly, using lots of space. Each question is worth 3 points.

Section 3.10 Linear approximations, differentials

(1) Let $f(x) = \sqrt[3]{x}$.

   (a) Find the linear approximation $L(x)$ to $f(x)$ at $x = 8$.

   (b) Use this linear approximation to estimate: $\sqrt[3]{8.2}$

   (c) Use you calculator to compute $\sqrt[3]{8.2}$ correct to 7 decimal places.

   (The linear approximation of $f(x)$ at $x = a$ is given by $L(x) = f(a) + f'(a)(x - a)$.)

(2) Use a linear approximation to estimate: 3.01^4

   (Hint: Use the linear approximation of $f(x) = x^4$ at 3.)

(3) Find the linear approximation to $\ln(x)$ at e^3.

(4) Use a linear approximation to estimate: $\dfrac{1}{4.975}$

(5) Let $y = \sqrt{1 + x^3}$.

   (a) Compute the differential dy.

   (b) Suppose $x = 2$ and then x changes to 2.07. Use the differential in part (a) to approximate the corresponding change in y.

Section 3.11 Hyperbolic functions

(6) Give the definitions of the hyperbolic functions: $\sinh x, \ \cosh x, \ \tanh x$

(7) Compute exactly (answers are fractions)

   (a) $\sinh(\ln 3)$

   (b) $\cosh(-\ln 3)$

   (c) $\tanh(\ln 3)$

(8) Use the definitions and algebra to prove the identity:

$$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$$

(9) Compute: **(a)** $\dfrac{d}{dx}(\ln x \,\cosh x)$ **(b)** $\dfrac{d}{dx}\tanh(3x^6)$

(10) Compute: **(a)** $\dfrac{d}{dx}\sinh^3 x$ **(b)** $\dfrac{d}{dx}\left(\dfrac{\sinh x}{1+\cosh x}\right)$

(11) The inverse hyperbolic sine function has the formula $\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$. Use this formula to find $\dfrac{d}{dx}\sinh^{-1} x$. Simplify your answer so that x appears only once in it.

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.

- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).

- Ask me about it after class.

- Come to my office hours: Mon 11:30 - 12:30, Wed 11:30 - 12:30 in CP 317.

- Go to the Math Tutorial Lab in-person in CP 303 or online.