Mth 31, Homework 6 on sections 3.7, 3.9

Due by Wed, Oct 22.

Write all your working out and answers neatly, using lots of space, and showing your work clearly. Each question is worth 3 points.

Section 3.7 Rates of change in natural sciences

(1) A tank has water draining out of it. If the volume V in gallons of water left in the tank after t minutes is

$$V = 2000\left(1 - \frac{t}{50}\right)^2$$

(a) Find the volume of water left after 40 minutes.

(b) At what rate is the water draining out of the tank at that time.

(c) When is the tank empty?

(Give the correct units for parts (a), (b) and (c).)

(2) A ball is thrown vertically up and its height after t seconds is

$$s(t) = 48t - 4t^2 \quad \text{meters.}$$

(a) Using the differentiation formulas, find the velocity of the ball: $v = s'$

(b) When does the ball have zero velocity?

(c) Find the maximum height of the ball.

(d) Find the velocity of the ball as it hits the ground.

(e) Find the acceleration of the ball: $a = v'$

(3) A bakery makes loaves of bread. Their cost to produce x loaves is

$$C(x) = 1000 + \frac{x}{4} + \frac{x^2}{1000} \text{ dollars.}$$

(a) Compute $C(100)$ and say what it means.

(b) Find the marginal cost. (This is $C'(x)$.)

(c) Compute $C'(100)$ and say what it estimates.

Section 3.9 Related rates

(4) The volume V of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$. If the radius of a sphere is increasing slowly at a rate of 2 cm per hour, find the rate of change of its volume when $r = 10$ cm. Write your answer in a sentence with the correct units.

(5) The length of a rectangle is increasing at 4 cm/s and its width is increasing at 3 cm/s. Find the rate of change of the area of the rectangle when the length is 20 cm and the width is 10 cm.

(6) Suppose $x^3 + y^3 = 28$. If $\frac{dx}{dt} = 4$ then find $\frac{dy}{dt}$ when $x = 1$ and $y = 3$.

(7) A 10 foot ladder is resting against a wall but the bottom is sliding out at 3 ft/min. How fast is the top of the ladder moving down when the bottom of the ladder is 8 feet from the wall?

(8) Two cars start from the same place, one going south at 50 mi/h and the other going east at 40 mi/h. What is the rate of change of the distance between the cars 3 hours later?

(9) *(Optional, for extra credit)* A plane is flying horizontally at a height of 1 mile and speed of 500 mi/h. It passes directly over a radar station. Find the rate at which the distance of the plane from the radar is increasing when this distance is 2 miles. Can you explain why this rate is less than 500 mi/h?

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.
- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).
- Ask me about it after class.
- Come to my office hours: Mon 11:30 - 12:30, Wed 11:30 - 12:30 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.