Mth 31, Homework 5 on sections 3.4, 3.5, 3.6

Due by Wed, Oct 15.

Write all your working out and answers neatly, using lots of space, and showing your work clearly. Each question is worth 3 points.

Section 3.4 The chain rule

(1) Name the rules and give the formulas needed for:

(a) $\frac{d}{dx}\Big(f(x)g(x)\Big)$ **(b)** $\frac{d}{dx}f(g(x))$ **(c)** $\frac{d}{dx}\frac{f(x)}{g(x)}$

(2) Define y as a function of x by

$$y = (3x+8)^4$$

(a) Express y as a composition of functions $y = f(g(x))$ by finding the inside function $g(x)$ and the outside function $f(x)$.

(b) Now use the chain rule to compute $\frac{dy}{dx}$

(3) Use the chain rule to find: **(a)** $\frac{d}{dx}\cos\left(\frac{1}{x^6}\right)$ **(b)** $\frac{d}{d\theta}e^{\tan\theta}$

(4) Compute $f'(x)$ for

$$f(x) = \left(\frac{x-2}{x+2}\right)^{30}$$

(5) Find the equation of the tangent line to the curve

$$y = \sqrt{1+x^3} \qquad \text{at the point} \qquad (2,3)$$

(6) For these exponential functions find: **(a)** $\frac{d}{dx}4^x$ **(b)** $\frac{d}{dx}4^{\sqrt{x}}$

(7) Compute the derivative of: $\sin(\tan(x^9+1))$

Section 3.5 Implicit differentiation

(8) The curve

$$x^4 + 3x^2y^2 - y^3 = 5$$

implicitly defines y as a function of x. Show why $\frac{dy}{dx} = \frac{4x^3+6xy^2}{3y^2-6x^2y}$

(9) Calculate $\frac{dy}{dt}$ by implicit differentiation if

$$te^y = 3t - y$$

(10) Find y' for the curve $\sin y + \cos x = \tan x$

(The notation y' means the first derivative $\frac{dy}{dx}$)

(11) For the curve

$$x^3 + xy + y^3 = 11$$

(a) Find y'

(b) Find the slope of the tangent line at $(1, 2)$.

(12) Find y' for the curve

$$\sin(xy) = \cos(x + y)$$

(Hint: you'll need the chain rule and the product rule here.)

Section 3.6 Derivatives of logs, inverse trig functions

For the next questions, remember our latest useful differentiation formulas:

$$\frac{d}{dx}b^x = (\ln b)b^x, \qquad \frac{d}{dx}\log_b x = \frac{1}{x\ln b},$$
$$\frac{d}{dx}\ln x = \frac{1}{x} \quad (\text{for } x > 0), \qquad \frac{d}{dx}\ln|x| = \frac{1}{x} \quad (\text{for } x \neq 0),$$
$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}, \qquad \frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}, \qquad \frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$$

Here, $\sin^{-1} x$ means the inverse $\sin$ function also called $\arcsin x$. Same for $\cos^{-1} x$, $\tan^{-1} x$.

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(13) Differentiate: $f(x) = x\ln x - x$

(14) Differentiate: **(a)** $g(x) = \log_{10}(x^6 + 1)$ **(b)** $F(t) = \sqrt{t + \ln|t|}$

(15) Use logarithmic differentiation to find $\frac{dy}{dx}$ for $y = \frac{x^{20}\sin^8(x)}{e^x + 1}$

(For logarithmic differentiation there are three steps: (A) take the natural log of both sides, (B) use properties of logs to expand products, quotients and powers, (C) now apply $\frac{d}{dx}$ to everything and use that $\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$.)

(16) Use logarithmic differentiation to differentiate: $x^{\cos x}$

(17) Differentiate: $f(x) = 2^\pi - \arcsin(2x + 1)$

(18) Differentiate: $g(x) = 6(\arctan x)^6$

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.
- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).
- Ask me about it after class.
- Come to my office hours: Mon 11:30 - 12:30, Wed 11:30 - 12:30 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.