Mth 31, Homework 2 on sections 2.5, 2.6, 2.7

Due by Wed, Sept 17.

Try these questions. Write all your working out and answers neatly by hand on your own notepaper and hand them to me next week. Please use lots of space and as many pages as you want, so I can include corrections or comments - otherwise I may ask you to redo it. It must be your own note paper, not a printout of this. You do not need to write the questions, but it is very important that you show clearly any work you had to do to get your answers. Each question is worth 3 points.

Section 2.5 Continuity

(1) Use the limit laws and the definition of continuity to explain why $f(x) = 3x + 1$ is continuous at $x = 4$.

(2) Draw three graphs to display the three types of discontinuities: removable discontinuity, jump discontinuity and infinite discontinuity.

(3) Let $f(x)$ be the function defined by

$$f(x) = \begin{cases} 1 - x & \text{if } x < 1 \\ x^2 - 1 & \text{if } x \geq 1. \end{cases}$$

(a) Sketch the graph of $f(x)$.

(b) Does $\lim_{x\to 1} f(x)$ exist?

(c) Explain if $f(x)$ is continuous or not at $x = 1$.

(4) As we saw in class, a theorem tells us that all the Mth 30 functions are continuous on their domains: polynomials, rational functions, root functions, exponential functions, logs, trigonometric and inverse trigonometric functions. We also saw that adding, subtracting, multiplying, dividing (bottom not 0), and composing continuous functions gives new continuous functions.

Use continuity to find:

(a)

$$\lim_{x\to 3} 2^x \sin x$$

(b)

$$\lim_{x\to 4} \frac{\ln(x^2)}{3x + \sqrt{x}}$$

(5) Give the intervals (using interval notation) where these functions are continuous:

(a) $g(x) = x + \ln(x) + \sqrt{x}$

(b) $h(x) = \dfrac{\sin x + \cos x}{x}$

(c) the function $f(x)$ from question **(3)**.

(6) Let $f(x) = x^3 + 3x^2 + 3x - 2$.

(a) Where is $f(x)$ continuous?

(b) Explain why the *Intermediate Value Theorem* shows that there is at least one solution to the equation

$$x^3 + 3x^2 + 3x - 2 = 0$$

in the interval $(0, 2)$.

Section 2.6 Limits at infinity; horizontal asymptotes

(7) Sketch the graph of an example of a continuous function f that satisfies all of these conditions:

$$\lim_{x\to-\infty} f(x) = 2, \quad f(-1) = 3, \quad f(2) = -2, \quad \lim_{x\to\infty} f(x) = -1.$$

(8) Find these limits or say they do not exist:

(a) $\displaystyle\lim_{x\to\infty} \frac{1}{x^2}$ **(b)** $\displaystyle\lim_{x\to\infty} \frac{1}{\sqrt{x}}$ **(c)** $\displaystyle\lim_{x\to\infty} e^{-x}$ **(d)** $\displaystyle\lim_{x\to\infty} 2x$ **(e)** $\displaystyle\lim_{x\to\infty} \sin x$

(9) Compute using algebra and the limit laws:

$$\lim_{x\to\infty} \frac{2x^2 + 1}{3x^2 - x}$$

(10) Compute using algebra and the limit laws:

$$\lim_{x\to\infty} \frac{\sqrt{x^2 + 4}}{2x + 5}$$

(11) Compute using algebra and the limit laws:

$$\lim_{x\to\infty} \sqrt{x^2 + 3} - x$$

Section 2.7 Derivatives and rates of change

(12) The derivative of a function $f(x)$ at $x = a$ can be defined as

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

as we saw in class.

Let $f(x) = 9x + 10$ and use this limit definition to calculate $f'(5)$.

(No points for any of these questions if you use derivative shortcuts – we'll get to those soon.)

(13) Let $g(x) = x^2 - 4x$.

(a) Find $g'(1)$ using the limit definition of the derivative.

(b) What is the slope of the tangent line to the graph of g at the point $(1, -3)$?

(c) Give the equation of this tangent line.

(14) The position of a rocket at time t, in seconds, is given by $s(t) = 2t^2 + 6$ measured in meters. Use the limit definition of derivative to find the rocket's velocity at $t = 10$.

(Hint: we want $s'(10)$ and state your answer with the correct units.)

(15) Use the limit definition to compute $f'(3)$ for $f(x) = \sqrt{2x + 10}$.

(16) Sketch a graph of $g(x)$ if you know that:

$$g(-2) = 3, \quad g'(-2) = -1, \quad g'(0) = 0, \quad g'(2) = 1, \quad g'(4) = 0$$

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.
- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).
- Ask me about it after class.
- Come to my office hours: Mon 11:30 - 12:30, Wed 11:30 - 12:30 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.