Mth 31, Homework 13 on sections 5.3, 5.4, 5.5

Extra Credit

Due by Wed, Dec 10.

Write all your working out and answers clearly and neatly, using lots of space. Each question is worth 3 points.

The Fundamental Theorem of Calculus

Suppose $f(x)$ is continuous for x in $[a, b]$. Then Part 1 of this theorem says

$$g'(x) = f(x) \qquad \text{if } g \text{ is given by} \qquad g(x) = \int_a^x f(t)\,dt.$$

Part 2 says

$$\int_a^b f(t)\,dt = F(b) - F(a) \qquad \text{for } F \text{ any antiderivative of } f.$$

Section 5.3 The Fundamental Theorem of Calculus

(1) Use part 2 of the theorem to calculate: $\displaystyle\int_0^{16} \sqrt{t}\,dt$

(2) Use part 2 to find: $\displaystyle\int_1^4 (4 - 2x)\,dx$

(Hint: use properties of integrals to break this up into simpler pieces and then find their antiderivatives.)

(3) Compute $\displaystyle\int_0^3 (4x^2 - 5)\,dx$ by using antiderivatives.

(4) Find: $\displaystyle\int_0^{3\pi} (1 + \cos\theta + 4e^{\theta})\,d\theta$

(5) Use part 2 to compute:

$$\int_1^4 \frac{2 + x^2}{\sqrt{x}}\,dx$$

(6) Is the evaluation $\displaystyle\int_{-2}^{2} \frac{3}{x^4}\,dx = -\frac{1}{4}$ correct? What could the problem be?

(7) Use part 1 of the theorem to find the derivative of

$$g(x) = \int_1^x \sqrt{t^2 + 1}\,dt$$

(8) Use part 1 and the chain rule to work out: $\dfrac{d}{dx}\displaystyle\int_0^{x^3} \sqrt{2t+1}\,dt$

Section 5.4 Indefinite integrals and the net change theorem

(9) Calculate the indefinite integral: $\displaystyle\int (12x^5+1)\,dx$

(This is another way of asking for the general antiderivative.)

(10) Find: $\displaystyle\int (\sin t - \sinh t)\ dt$

(11) Find: $\displaystyle\int (3u+2)(u+4)\,du$

(12) Compute the following definite integral by first finding the indefinite integral and then using Part 2 of the fundamental theorem:

$$\int_1^3 \frac{2x^2+4x-1}{x}\,dx$$

(13) Compute the definite integral: $\displaystyle\int_0^{\pi/4} \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta$

(14) An object moves in a straight line with position $s(t)$ cm and velocity $v(t)$ cm/s at time t. Suppose the object's acceleration is $a(t) = 2t - 1$ and $v(0) = -12$.

(a) Explain what $v(0) = -12$ means here in your own words.

(b) Find $v(t)$

(c) What is the displacement of the object between $t = 0$ and $t = 6$ seconds? (In other words, what is the net change of its position?)

(d) What is the total distance traveled by the object between $t = 0$ and $t = 6$ seconds?

Section 5.5 The substitution rule

(15) Find the indefinite integral $\displaystyle\int \cos(10x)\,dx$ by using the substitution $u = 10x$.

(16) Find the indefinite integral $\displaystyle\int \tan^2(\theta)\sec^2(\theta)\,d\theta$ by using $u = \tan(\theta)$.

(17) Use the substitution rule to find: $\displaystyle\int \frac{\cos(\sqrt{t})}{\sqrt{t}}\,dt$

Differentiate your answer and check you get $\dfrac{\cos(\sqrt{t})}{\sqrt{t}}$.

(18) Find: $\displaystyle\int \frac{e^x}{e^x + \pi}\, dx$

(19) Evaluate: $\displaystyle\int 2x^3\sqrt{x^2+3}\, dx$

(20) Compute the definite integral: $\displaystyle\int_{\sqrt{\pi}}^{\sqrt{2\pi}} 8x\sin(x^2)\, dx$

(21) Compute: $\displaystyle\int_{2/3}^{1} (3y-2)^{1000}\, dy$

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.
- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).
- Ask me about it after class.
- Come to my office hours: Mon 11:30 - 12:30, Wed 11:30 - 12:30 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.