Mth 31, Homework 12 on sections 5.1, 5.2

Due by Wed, Dec 3.

Write all your working out and answers clearly and neatly, using lots of space. Each question is worth 3 points.

Section 5.1 Areas and distance

(1) Let $f(x) = x^{-2}$. Use two rectangles and midpoints to estimate the area under this graph between $x = 1$ and $x = 5$. Draw a diagram showing the graph and rectangles.

(2) Let $f(x) = 2 + \sqrt{x}$. Use three rectangles and midpoints to estimate the area under this graph between $x = 0$ and $x = 3$. Draw a diagram showing the graph and rectangles.

(3) The velocity $v(t)$ of a car is measured every half hour, giving the following data (t is measured in hours and $v(t)$ measured in miles per hour):

$$v(0.5) = 55, \quad v(1) = 40, \quad v(1.5) = 50, \quad v(2) = 60, \quad v(2.5) = 65$$

Approximately how far did the car travel between $t = 0$ and $t = 2.5$?

(4) Write the exact area under the graph of $g(x) = x^3$ between $x = 1$ and $x = 4$ as a limit of rectangle areas using right endpoints. Do not evaluate the limit.

(For this we have $a = 1$ and $b = 4$. Then $\Delta x = (b - a)/n$. Also $x_i = a + i\Delta x$ gives the right endpoint of the ith rectangle base. The area is the limit as $n \to \infty$ of the rectangle areas.)

(5) Write the exact area under the graph of $f(x) = e^x$ between $x = 0$ and $x = 2$ as a limit of rectangle areas using right endpoints. Do not evaluate the limit.

Section 5.2 The definite integral

(6) Compute this sigma notation expression: $2\sum_{i=3}^{6}(i^2 - 3i)$

(7) Give the definition of the definite integral $\int_a^b f(x)\,dx$ as a Riemann sum limit, saying what Δx and x_i^* are.

(8) Write the following limit, on the interval $[0, 2\pi]$, as a definite integral:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\cos(x_i)}{x_i^2 + 1}\,\Delta x$$

(9) Recall the area in Question (2) under $f(x) = 2 + \sqrt{x}$ between $x = 0$ and $x = 3$.

(a) Write this area as a definite integral.

(b) Write this area as a limit using right endpoints. Do not evaluate the limit.

(10) Use a Riemann sum with $n = 2$ rectangles, taking the sample points to be midpoints, to estimate:

$$\int_0^4 \sqrt{x^3+1}\,dx$$

(11) Write the definition of $\displaystyle\int_1^4 (4-2x)\,dx$ as a limit of Riemann sums using right end points. Then use the formulas

$$\sum_{i=1}^{n} 1 = n, \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

to find this limit.

(12) Graph the line $y = 4 - 2x$. Use this graph and the areas of the triangles it makes to find $\int_1^4 (4-2x)\,dx$ in a second way.

(Don't forget that area below the x-axis counts negative.)

(13) Compute $\displaystyle\int_0^4 3x^2\,dx$ by using its definition as a limit of Riemann sums, using right end points, along with the formula

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

We saw these basic properties of definite integrals:

$$\int_a^a f(x)\,dx = 0 \qquad \int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx \qquad \int_a^b 1\,dx = b - a$$

$$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx \qquad \int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

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(14) Use these properties to find: **(a)** $\displaystyle\int_2^2 \tan(x)\,dx$ **(b)** $\displaystyle\int_{-2}^6 4\,dx$ **(c)** $\displaystyle\int_6^{-2} 4\,dx$

(15) Find

$$\int_0^{\pi/2} (3 + 8\sin^4 x)\,dx + \int_{\pi/2}^{\pi} (7 + 8\sin^4 x)\,dx$$

if we are given that $\displaystyle\int_0^{\pi} \sin^4 x\,dx = \frac{3\pi}{8}$.

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.
- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).
- Ask me about it after class.
- Come to my office hours: Mon 11:30 - 12:30, Wed 11:30 - 12:30 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.