Mth 31, Homework 10 on sections 4.5, 4.7

Due by Wed, Nov 19.

Write all your working out and answers clearly and neatly, using lots of space. Each question is worth 3 points.

Section 4.5 Summary of curve sketching

To sketch the graph of $f(x)$ we first work out the following:

(a) Find the domain of f.

(b) Find all x and y intercepts.

(c) Is f odd, even or periodic? (Odd means $f(-x) = -f(x)$, even means $f(-x) = f(x)$ and periodic means $f(x + P) = f(x)$.)

(d) Find all vertical and horizontal asymptotes.

(e) Give the intervals where f is increasing and decreasing.

(f) Find all the local maximums and minimums: identify which is which and give their coordinates.

(g) Find where f is concave up or down and locate any inflection points.

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(1) Let $f(x) = x^4 - 8x^2 + 8$. Work out its properties (a) – (g).

(2) Graph $f(x)$ from the previous question using the properties you found and plotting any extra points you need.

(3) Let $g(x) = \dfrac{x}{x^2 - 4}$. Work out its properties (a) – (g).

(4) Graph $g(x)$ from the previous question using the properties you found and plotting any extra points you need.

(5) Let $h(x) = x + \sin(x)$. Work out its properties (a) – (g).

(6) Graph $h(x)$ from the previous question using the properties you found and plotting any extra points you need.

Section 4.7 Optimization

(7) Find two numbers with sum 18 and product as large as possible using these steps:

(a) Let x and y be the two numbers and $P = xy$ their product. Write the equation for "two numbers with sum 18".

(b) Write P as a function of x only and compute $\frac{d}{dx}P$.

(c) Find when $\frac{d}{dx}P = 0$. Is this a maximum? Write "The two numbers are .... and ...."

(8) Find the point on the line $y = 1 + 2x$ that is closest to the point $(5, 1)$.

(Hint: minimize the square of the distance between the points.)

(9) Find the coordinates of the two points on the ellipse $3x^2 + y^2 = 28$ that are furthest from the point $(6, 0)$.

(10) A farmer wants to enclose an area of 6 square miles with a rectangular fence. The fence should also divide the area in two with a line of fence running parallel to two sides. What is the shortest length of fence the farmer can use? Draw a diagram of this shortest fence.

(11) A plastic bucket in the shape of a circular cylinder with a base but no top is being designed to contain a volume of $8000\pi\ cm^3$. What should the dimensions be to minimize the amount of plastic.

(Hint: if the bucket has base of radius r and height h then the area of the base circle is πr^2, the area of the sides is $h \cdot 2\pi r$ and the volume is $h \cdot \pi r^2$.)

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.
- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).
- Ask me about it after class.
- Come to my office hours: Mon 11:30 - 12:30, Wed 11:30 - 12:30 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.