

Mth 31, Homework 1 on sections 2.1, 2.2, 2.3

Due by Wed, Sept 10.

Here are 13 questions for you to try. Write all your working out and answers neatly by hand on your own notepaper and hand them to me next week. Please use lots of space and as many pages as you want, so I can include corrections or comments - otherwise I may ask you to redo it. It must be your own note paper, not a printout of this. You do not need to write the questions, but it is very important that you show clearly any work you had to do to get your answers. Each question is worth 3 points.

Section 2.1 Tangents and velocity

- (1) Draw lines with these slopes: (a) 2, (b) $1/3$, (c) -3
 - (2) The point $P(1, 1)$ is on the curve $y = x^3$.
 - (a) Find the slope of the secant line through $Q(2, 8)$ and $P(1, 1)$.
 - (b) Find the slope of the secant line through $R(1.01, 1.01^3)$ and $P(1, 1)$.
 - (c) Use the above work to estimate the slope of the tangent line to the curve $y = x^3$ at $P(1, 1)$.
(Hint: what number are your answers from parts (a) and (b) getting close to?)
 - (3) A phone is accidentally dropped out a window. Physics tells us that the distance it has fallen after t seconds is $s(t) = 4.9t^2$ meters. (We are calling the position function $s(t)$.)
 - (a) How far has the phone fallen after 2 seconds?
 - (b) Find the average velocity (in meters per second) of the phone between $t = 1$ second and $t = 2$ seconds. (Hint: average velocity = distance/time.)
 - (c) Find the average velocity of the phone between $t = 1.99$ seconds and $t = 2$ seconds.
 - (d) Use the above work to estimate the instantaneous velocity of the phone at $t = 2$ seconds.
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Section 2.2 Limit of a function

- (4) Explain in your own words what this means (don't use the word *limit* in your answer):

$$\lim_{x \rightarrow 4} f(x) = 6$$

(5) Suppose we know

$$\lim_{x \rightarrow 2^-} g(x) = 1, \quad g(2) = 3, \quad \lim_{x \rightarrow 2^+} g(x) = -1.$$

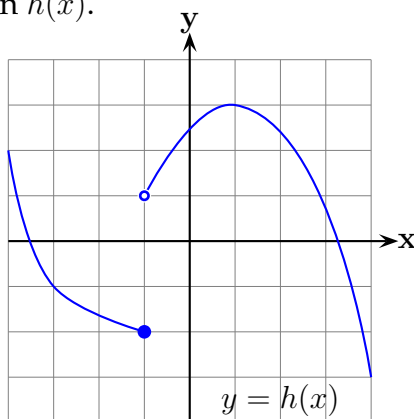
Sketch what the graph of $g(x)$ could look like. Use open circles for missing points and closed circles for points that are there.

(6) Numerically estimate

$$\lim_{x \rightarrow 0^+} x^x$$

(Hint: this is x to the power x . Use your calculator to calculate this for $x = 0.1$, $x = 0.01$ and so on until you think you see the limit.)

(7) This is the graph of the function $h(x)$.



Find the following:

(a) $h(1)$

(b) $h(-1)$

(c) $\lim_{x \rightarrow -1^-} h(x)$ (left-sided limit)

(d) $\lim_{x \rightarrow -1^+} h(x)$ (right-sided limit)

(e) $\lim_{x \rightarrow -1} h(x)$ (two-sided limit)

(8) Find:

$$\lim_{x \rightarrow 2^+} \frac{x-3}{x-2}$$

(Hint: your answer should be ∞ or $-\infty$.)

Section 2.3 Calculating limits using limit laws

(9) Suppose

$$\lim_{x \rightarrow 0} f(x) = 3, \quad \lim_{x \rightarrow 0} g(x) = 2, \quad \lim_{x \rightarrow 2} f(x) = -1, \quad \lim_{x \rightarrow 2} g(x) = 0.$$

Use the limit laws to calculate

(a) $\lim_{x \rightarrow 0} ((f(x))^2 + 4g(x))$

(b) $\lim_{x \rightarrow 2} \frac{g(x) + 4}{f(x) - 3}$

(10) Find

$$\lim_{x \rightarrow 3} (x^2 - x + 2)$$

using the limit laws. Say which law you are using at each step.

(11) Compute these limits using algebra to simplify and the limit laws:

(a) $\lim_{h \rightarrow 0} \frac{(h + 4)^2 - 16}{4h}$

(b) $\lim_{x \rightarrow -2} \frac{\frac{1}{x} + \frac{1}{2}}{x + 2}$

(Remember: to add $\frac{1}{x} + \frac{1}{2}$ we need a common denominator = $\frac{2}{2x} + \frac{x}{2x} = \dots$)

(12) Compute this limit using algebra and the limit laws:

$$\lim_{h \rightarrow 0} \frac{\sqrt{h + 4} - 2}{3h}$$

(Hint: use the conjugate as we did in class.)

(13) Find this limit using the *Squeeze Theorem* and the limit laws:

$$\lim_{x \rightarrow 0} x^2 \cos(1/x)$$

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.
- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).
- Ask me about it after class.
- Come to my office hours: Mon 11:30 - 12:30, Wed 11:30 - 12:30 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.