Mth 21, Homework 2 on sections 2.3, 2.4, 2.5

Due by Mon, Sept 29.

Write all your working out and answers neatly by hand on your own notepaper and hand them to me by the date shown. Please use lots of space. You do not need to write the questions, but it is very important that you show clearly any work you had to do to get your answers. Each question is worth 3 points.

Section 2.3 Introduction to Combinatorics

(1) In a certain sandwich shop you much choose from 4 types of bread, 5 types of filling, 3 kinds of sauces and having it hot or cold. How many different kinds of sandwiches is it possible to order?

(Hint: You can use the boxes method from the Fundamental Principle of Counting.)

(2) A serial number consists of two letters followed by an X or a Y and then three digits with no digit repeated. For example, a possible serial number here is TQY274. What is the total number of possible serial numbers?

(3) Forty women run in the olympic marathon final. How many ways can the gold, silver and bronze medals be awarded?

(4) Compute these factorial expressions: **(a)** $5!$ **(b)** $\dfrac{30!}{27! \cdot 3!}$

Section 2.4 Permutations and Combinations

(5) Compute ${}_7P_3$ which is the number of ways to arrange 3 things selected from 7.

(Hint: You can use the formula ${}_nP_r = n!/(n-r)!$ or the Fundamental Principle of Counting.)

(6) How many ways can you arrange the six books on your small bookshelf?

(7) **(a)** Compute the combination number ${}_9C_4$ using its factorial formula.

(b) Explain what this number counts in your own words.

(c) Give an example of a question that would have this number as an answer.

(8) How many ways are there to select 3 people from 20 job applicants if

(a) the order of selection is not important,

(b) the order of selection is important?

(Hint: Can you see which one needs permutations and which needs combinations?)

Section 2.5 Infinite sets

(9) Two sets are equivalent if there is a one-to-one correspondence between them.

(a) Find $n(S)$, the cardinality of

$$S = \{3, 6, 9, \ldots, 171, 174\}$$

by setting up a correspondance with some of the counting numbers $\{1, 2, 3, \ldots\}$.

(b) The sets A and B are given by

$$A = \{2, 4, 6, 8, \ldots, 362\}$$
$$B = \{10, 20, 30, 40, \ldots, 1800\}.$$

Are these sets equivalent? Explain.

(10) The infinite sets N of natural (counting) numbers, and O of positive odd numbers are

$$N = \{1, 2, 3, 4, \ldots\}$$
$$O = \{1, 3, 5, 7, \ldots\}.$$

(a) Are N and O equivalent? Explain.

(b) If they are equivalent, which element of N corresponds to $37 \in O$?

(c) What does countable mean exactly?

(d) Is O countable or uncountable?

(11) We saw in class that the following table can be used to show that all the positive fractions are countable:

$$\begin{array}{ccccccc}
\frac{1}{1} & \frac{2}{1} & \frac{3}{1} & \frac{4}{1} & \frac{5}{1} & \frac{6}{1} & \ldots \\
\frac{1}{2} & \frac{2}{2} & \frac{3}{2} & \frac{4}{2} & \frac{5}{2} & \frac{6}{2} & \ldots \\
\frac{1}{3} & \frac{2}{3} & \frac{3}{3} & \frac{4}{3} & \frac{5}{3} & \frac{6}{3} & \ldots \\
\frac{1}{4} & \frac{2}{4} & \frac{3}{4} & \frac{4}{4} & \frac{5}{4} & \frac{6}{4} & \ldots \\
\frac{1}{5} & \frac{2}{5} & \frac{3}{5} & \frac{4}{5} & \frac{5}{5} & \frac{6}{5} & \ldots \\
\ldots\ldots & & & & & &
\end{array}$$

Use the diagonal pattern we saw to continue this list of fractions $\frac{1}{1}, \frac{1}{2}, \frac{2}{1}, \frac{3}{1}, \frac{1}{3}$ as far as $\frac{1}{5}$. (Don't forget to leave out any fractions like $\frac{2}{2}$ that reduce to an earlier fraction.)

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.
- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).
- Ask me about it after class.
- Come to my office hours: Mon 11:30 - 12:30, Wed 11:30 - 12:30 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.