Mth 21, Homework 11 on section 12.1

Will not be collected.

Try these questions and see if you get the correct answers on the back. This set will not be collected. A question like these will be on the final exam.

Section 12.1 The geometry of linear programming

(1) **(a)** Graph the triangular region bounded by the inequalities:

$$\begin{aligned} x &\geqslant 0 \\ y &\geqslant 0 \\ 2x + y &\leqslant 2 \end{aligned}$$

(b) Give the coordinates of its three corner points.

(2) Maximize the objective function $z = 3x + 2y$ subject to the constraints in question 1.

(Hint: the key idea of linear programming is that any maximum (or minimum) must be at a corner point. So work out z at each of the three corner points you found and use the biggest.)

(3) A carpenter makes x tables and y chairs from a supply of 200 kilograms of wood every week. Suppose she has time to make at most 12 items per week, and each table needs 30 kg and each chair 10 kg of wood. Give the 4 inequalities that describe this situation.

(4) Maximize $z = x + y$ subject to the constraints:

$$\begin{aligned} x &\geqslant 0 \\ y &\geqslant 0 \\ x + 3y &\leqslant 9 \\ 2x + y &\leqslant 8 \end{aligned}$$

(5) Minimize $z = 4x + 3y$ subject to the constraints:

$$\begin{aligned} x &\geqslant 0 \\ y &\geqslant 0 \\ x + y &\geqslant 4 \\ x + 2y &\geqslant 6 \end{aligned}$$

(Hint: graph the corresponding region - it is infinitely large and does not have $(0, 0)$ as a corner. Since we are minimizing, find the corner point that makes z smallest.)

Solutions.

(1) Draw the x, y axes and the triangle with the three corner points at $(0, 0)$, $(0, 2)$, $(1, 0)$.

(2) At these corner points the objective function is $0, 4, 3$, respectively. So the maximum is 4.

(3) The carpenter's 4 inequalities are:

$$\begin{aligned} x &\geqslant 0 \\ y &\geqslant 0 \\ x + y &\leqslant 12 \\ 30x + 10y &\leqslant 200 \end{aligned}$$

(4) At the three corner points the objective function is $3, 4, 5$. So the maximum is 5.

(5) At the three corner points the objective function is $12, 14, 24$. So the minimum is 12.

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.
- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).
- Ask me about it after class.
- Come to my office hours: Mon 11:30 - 12:30, Wed 11:30 - 12:30 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.