MTH 31 SAMPLE FINAL EXAMINATION
... 3 HOURS

INSTRUCTIONS: Do any 15 of these 18 equal value questions. To get maximum points it is important that you show clearly all your working out and reasoning. You may use your own calculator or mine only. Phones and other technology must be put away.

Here are 18 questions from the homework sets, or very similar, to give you an idea of what to expect on the actual final and to see its format. Any homework question could appear.

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(1) Let $f(x)$ be the function defined by

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ x+1 & \text{if } x > 1. \end{cases}$$

(a) Sketch the graph of $f(x)$.

(b) Does $\lim_{x\to 1} f(x)$ exist? Explain.

(c) Explain why $f(x)$ is continuous or not at $x = 1$.

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(2) Compute these limits exactly:

(a) $\displaystyle\lim_{x\to 0} \frac{-1+\cos(2x)}{x^2}$ **(b)** $\displaystyle\lim_{x\to 0} \frac{e^x - 2^x}{4x}$ **(c)** $\displaystyle\lim_{x\to\infty} \frac{\sqrt{x}-4}{x-4}$

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(3) Let $f(x) = x^3 - 4x - 2$.

(a) Explain why the *Intermediate Value Theorem* shows that there is at least one solution to the equation $f(x) = 0$ in the interval $(0, 3)$.

(b) What exactly does the *Mean Value Theorem* say about $f(x)$ on the interval $[0, 3]$?

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(4) Let $g(x) = x^2 - 3x + 1$ and use the limit definition of derivative to find $g'(x)$

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(5) Compute these derivatives:

(a) $\displaystyle\frac{d}{dt} \ln(t)\arcsin(t)$ **(b)** $\displaystyle\frac{d}{dx}\frac{\sinh(x)}{x^2+1}$ **(c)** $\displaystyle\frac{d}{dx}\log_{10}(x^6+3)$

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(6) Find the equation of the tangent line at $(1, 2)$ to the curve

$$x^3 + 3xy + y^3 = 15$$

(7) A ball is thrown vertically up and its height after t seconds is $s(t) = 48t - 4t^2$ meters. Answer these questions using the correct units.

(a) Find the velocity of the ball: $v(t)$

(b) Find the acceleration of the ball: $a(t)$

(c) When does the ball have zero velocity?

(d) Find the maximum height of the ball.

(e) Find the velocity of the ball as it hits the ground.

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(8) A 20 foot ladder is resting against a wall but the bottom is sliding out at 2 ft/min. How fast is the top of the ladder moving down when the bottom of the ladder is 12 feet from the wall?

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(9) Find the absolute maximum and minimum values on the interval $[-2, 3]$ of the function $g(x) = 2x^3 - 3x^2 - 12x + 1$.

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(10) Let $f(x) = x^3 - 3x^2 + 1$.

(a) Give the intervals where f is increasing/decreasing.

(b) Find all local maximums and minimums: identify which is which and give their coordinates.

(c) Give the intervals where f is concave up/down and locate any inflection points.

(d) Graph f by using all the information you have found and plotting any extra points you need.

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(11) Use logarithmic differentiation to find $\dfrac{dy}{dx}$ for $y = \dfrac{x^4 \sin^3 x}{e^x + 1}$

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(12) Use a linear approximation to estimate: 2.01^4

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(13) A farmer wants to enclose an area of 6 square kilometers with a rectangular fence. The fence should also divide the area in two with a line of fence running parallel to two sides. What is the shortest length of fence the farmer can use? Draw a diagram of this shortest fence arrangement.

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(14) Use Newton's method to estimate $\sqrt[3]{25}$. Start with an initial x_1 and apply Newton's method twice to obtain x_2 and x_3.

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(15) Use a Riemann sum with $n = 3$ rectangles, taking the sample points to be midpoints, to estimate:

$$\int_2^{14} \ln(x-1)\,dx$$

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(16) By using its definition as a limit of Riemann sums, compute $\displaystyle\int_1^3 (2x^2)\,dx$ using right end points. Recall the formulas you might need:

$$\sum_{i=1}^{n} 1 = n, \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$

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(17) Use the two parts of the *Fundamental Theorem of Calculus* to:

(a) Find $\displaystyle\frac{d}{dx}\int_3^{x^2} \cos(t^3)\,dt$

(b) Compute $\displaystyle\int_0^2 (3x^2 - 4x)\,dx$

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(18) Evaluate these definite and indefinite integrals:

(a) $\displaystyle\int_1^4 \frac{1-\sqrt{t}}{t^2}\,dt$

(b) $\displaystyle\int_0^1 \frac{e^x}{e^x+2}\,dx$

(c) $\displaystyle\int x^3\sqrt{x^2-1}\,dx$