Mth 31, Homework 8 on sections 4.2, 4.3

Due by Wed, Nov 6.

Try these questions. Write all your working out and answers by hand on your own notepaper and hand them to me next week. Please use lots of space and as many pages as you want, so I can include corrections or comments. You do not need to write the questions, but it is very important that you show clearly any work you had to do to get your answers. Each question is worth 3 points.

Section 4.2 The Mean Value Theorem

(1) Your flight from New York to Los Angeles takes 6 hours to go 2400 miles. What is your average velocity? What does the *Mean Value Theorem* say about your flight?

(2) Let $f(x) = -2x^2 + x + 1$.

- **(a)** What exactly does the Mean Value Theorem say about $f(x)$ on the interval $[-1, 3]$?
- **(b)** Find all the numbers c that satisfy the conclusion of the theorem.

(3) Let $g(x) = x^3 - 3x + 2$.

- **(a)** What exactly does the Mean Value Theorem say about $g(x)$ on the interval $[-2, 2]$?
- **(b)** Find all the numbers c that satisfy the conclusion of the theorem.

(4) Let $h(x) = 2 + \ln x$.

- **(a)** What exactly does the Mean Value Theorem say about $h(x)$ on the interval $[1, 10]$?
- **(b)** Find all the numbers c that satisfy the conclusion of the theorem.

(5) Suppose $f(2) = -3$ and $f'(x) \geqslant 3/2$ for $2 \leqslant x \leqslant 6$. Use the Mean Value Theorem to find the smallest possible value of $f(6)$.

(6) Show that the equation

$$x^3 - 12x + 3 = 0$$

has exactly one solution in the interval $[-1, 1]$ with the following steps.

- **(a)** Use the Intermediate Value Theorem for $f(x) = x^3 - 12x + 3$ on $[-1, 1]$ to show that there is at least one solution in the interval.
- **(b)** Now suppose there are two solutions a and b in the interval with $a < b$. In other words $f(a) = 0$ and $f(b) = 0$. Apply the Mean Value Theorem to $f(x)$ on $[a, b]$. What does it say?
- **(c)** Explain why what is says cannot be true. This means there cannot be two solutions – there is exactly one (though we don't know what it is).

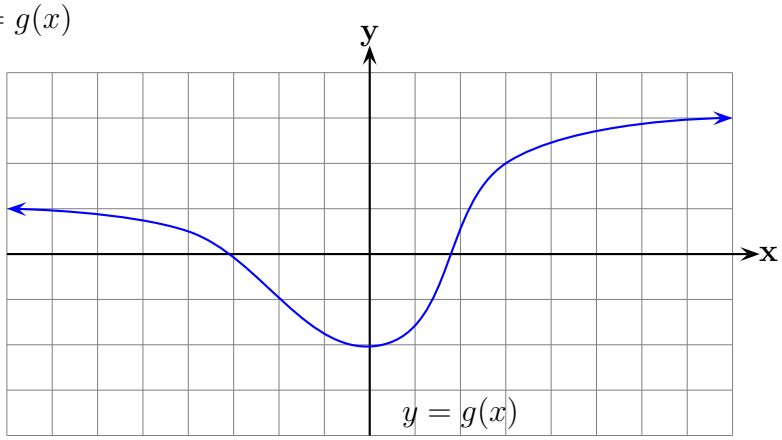
Section 4.3 How derivatives affect graph shape

(7) Let $f(x) = -x^2 + 2x + 15$.

(a) Use the first derivative to see the xs where f is increasing and decreasing. Give your answer in interval notation.

(b) Find all the local maximums and minimums: identify which is which and give their coordinates.

(8) For this graph $y = g(x)$



(a) Give the intervals where g is increasing and decreasing.

(b) Identify and locate all local maximums and minimums.

(c) Give the intervals where g is concave up and down.

(d) Locate all inflection points.

(9) Let $f(x) = x^3 - 12x^2 - x$.

(a) Use the second derivative to see the xs where f is concave up and down. Give your answers in interval notation.

(b) Locate all inflection points.

(10) For the graph of $\sin x$ on the interval $[0, 2\pi]$, find the intervals where it is increasing/decreasing and where it is concave up/down.

(11) Let $G(x) = \ln(x^2 + 4)$. Sketch its graph after finding the following.

(a) Give the intervals where G is increasing and decreasing.

(b) Identify and locate all local maximums and minimums.

(c) Give the intervals where G is concave up and down.

(d) Locate all inflection points.

(12) The latest unemployment numbers show that the number of unemployed people is increasing, but at a decreasing rate. Say what this statement means in terms of the derivatives of a function. (Think of $U(t)$ as the number unemployed at time t.)

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.
- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).
- Ask me about it after class.
- Come to my office hours: Mon 12:00 - 1:00, Wed 12:00 - 1:00 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.