Mth 31, Homework 7 on sections 3.10, 3.11, 4.1

Due by Wed, Oct 30.

Try these questions. Write all your working out and answers by hand on your own notepaper and hand them to me next week. Please use lots of space and as many pages as you want, so I can include corrections or comments. You do not need to write the questions, but it is very important that you show clearly any work you had to do to get your answers. Each question is worth 3 points.

Section 3.10 Linear approximations, differentials

(1) Let $f(x) = \sqrt[3]{x+1}$.

- **(a)** Find the linear approximation to $f(x)$ at 0.
- **(b)** Use this linear approximation to estimate: $\sqrt[3]{1.06}$
- **(c)** Use you calculator to compute $\sqrt[3]{1.06}$ correct to 5 decimal places.

(2) Use a linear approximation to estimate 2.01^4.
(Hint: Use the linear approximation of x^4 at 2.)

(3) Find the linear approximation to $\ln(x)$ at e^3.

(4) Use a linear approximation to estimate: $\dfrac{1}{4.975}$

(5) Let $y = \sqrt{1+x^3}$.

- **(a)** Compute the differential dy.
- **(b)** Suppose $x = 2$ and then x changes to 2.07. Use the differential in part (a) to approximate the corresponding change in y.

Section 3.11 Hyperbolic functions

(6) Give the definitions of the hyperbolic functions: $\sinh x,\ \cosh x,\ \tanh x$

(7) Compute exactly (answers are fractions)

- **(a)** $\sinh(\ln 3)$
- **(b)** $\cosh(-\ln 3)$
- **(c)** $\tanh(\ln 3)$

(8) Use the definitions and algebra to prove the identity:

$$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$$

(9) Compute: **(a)** $\dfrac{d}{dx}(\ln x \ \cosh x)$ **(b)** $\dfrac{d}{dx}\tanh(3x^6)$

(10) Compute: **(a)** $\dfrac{d}{dx}\sinh^3 x$ **(b)** $\dfrac{d}{dx}\left(\dfrac{\sinh x}{1+\cosh x}\right)$

(11) The inverse hyperbolic sine function has the formula $\sinh^{-1} x = \ln\left(x+\sqrt{x^2+1}\right)$. Use this formula to find $\dfrac{d}{dx}\sinh^{-1} x$. Simplify your answer so that x appears only once in it.

Section 4.1 Maximum and minimum values

(12) Let $f(x) = x^3 - 2x - 4$. Find the values of this function

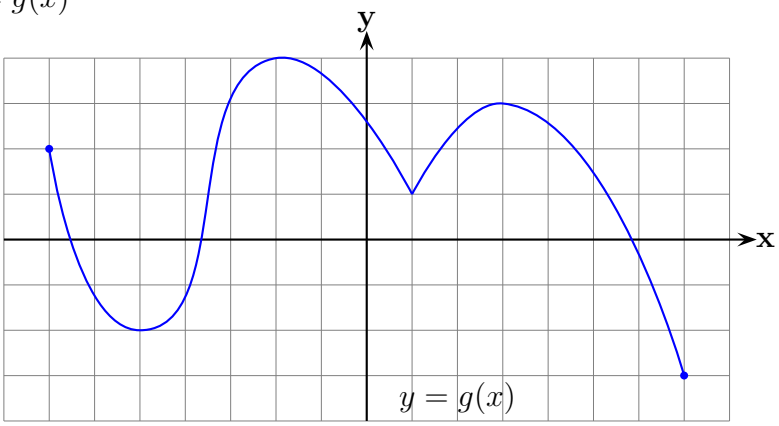
(a) at $x = 2$

(b) at $x = -3$

(c) at $x = 1/2 = 0.5$

(13) Which of the numbers $-2,\ 1.4,\ -1,\ 6,\ 3.001,\ -0.99$ are in the closed interval $[-1, 3]$?

(14) For this graph $y = g(x)$



(a) Identify all the local maximums and minimums of $g(x)$ and give the coordinates of these points (there are 4 of them). Remember that end-points are never called local maximums or minimums.

(b) Give the values of g at these local maximums and minimums (these are the ys).

(c) Identify the absolute maximum and minimum of $g(x)$ and give the coordinates of these points. (Can be end-points.)

(d) Give the absolute maximum and minimum values of g.

(15) Sketch the graph of $f(x) = x^2 - 1$ for x in the closed interval $[-1, 2]$. Use your graph to find the absolute maximum and minimum values of the function on this interval and identify all local maximums and minimums.

(16) Remenber that a *critical number* for $f(x)$ is a c where $f'(c) = 0$ or $f'(c)$ does not exist.

(a) Find all the critical numbers of $g(x)$ in question (14). These critical numbers are xs.

(b) Find the critical numbers of: $F(x) = x^3 - 27x + 9$

(c) Find the critical numbers of: $G(x) = |3x + 12|$

(17) Use the closed interval method to find the absolute maximum and minimum values of $f(x) = 2x^3 - 3x^2 - 12x + 1$ on $[-2, 3]$.

(18) Use the closed interval method to find the absolute maximum and minimum of

$$h(x) = \frac{x}{x^2 + 1} \quad \text{on} \quad [0, 3]$$

(Hint: when you look at $h'(x)$ remember that the only way for a rational function to be zero is if the numerator (top) is zero.)

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.
- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).
- Ask me about it after class.
- Come to my office hours: Mon 12:00 - 1:00, Wed 12:00 - 1:00 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.