Mth 31, Homework 2 on sections 2.3, 2.5, 2.6

Due by Wed, Sept 18.

Try these 13 questions. Write all your working out and answers by hand on your own notepaper and hand them to me next week. Please use lots of space and as many pages as you want, so I can include corrections or comments - otherwise I will ask you to redo it. It must be your own note paper, not a printout of this. You do not need to write the questions, but it is very important that you show clearly any work you had to do to get your answers. Each question is worth 3 points.

Section 2.3 Calculating limits using limit laws

(1) Compute this limit using algebra and the limit laws:

$$\lim_{h\to 0} \frac{\sqrt{h+4}-2}{3h}$$

(Hint: use the conjugate as we did in class. This useful trick is also needed in question **(13)** below.)

(2) Find this limit using the *Squeeze Theorem* and the limit laws:

$$\lim_{x\to 0} x^2 \cos(1/x)$$

Section 2.5 Continuity

(3) Use the limit laws and the definition of continuity to explain why $f(x) = 3x + 1$ is continuous at $x = 4$.

(4) Draw three graphs to display the three types of discontinuities: removable discontinuity, jump discontinuity and infinite discontinuity.

(5) Let $f(x)$ be the function defined by

$$f(x) = \begin{cases} 1 - x & \text{if } x < 1 \\ x^2 - 1 & \text{if } x \geq 1. \end{cases}$$

(a) Sketch the graph of $f(x)$.

(b) Does $\lim_{x\to 1} f(x)$ exist?

(c) Explain if $f(x)$ is continuous or not at $x = 1$.

(6) As we saw in class, a theorem tells us that all the Mth 30 functions are continuous on their domains: polynomials, rational functions, root functions, exponential functions, logs, trigonometric and inverse trigonometric functions. We also saw that adding, subtracting, multiplying, dividing (bottom not 0), and composing continuous functions gives new continuous functions.

Use continuity to find:

(a)

$$\lim_{x\to 3} 2^x \sin x$$

(b)

$$\lim_{x\to 4} \frac{\ln(x^2)}{3x+\sqrt{x}}$$

(7) Give the intervals (using interval notation) where these functions are continuous:

(a) $g(x) = x + \ln(x) + \sqrt{x}$

(b) $h(x) = \dfrac{\sin x + \cos x}{x}$

(c) the function $f(x)$ from question **(5)**.

(8) Let $f(x) = x^3 + 3x^2 + 3x - 2$.

(a) Where is $f(x)$ continuous?

(b) Explain why the *Intermediate Value Theorem* shows that there is at least one solution to the equation

$$x^3 + 3x^2 + 3x - 2 = 0$$

in the interval $(0, 2)$.

Section 2.6 Limits at infinity; horizontal asymptotes

(9) Sketch the graph of an example of a continuous function f that satisfies all of these conditions:

$$\lim_{x\to-\infty} f(x) = 2, \quad f(-1) = 3, \quad f(2) = -2, \quad \lim_{x\to\infty} f(x) = -1.$$

(10) Find these limits or say they do not exist:

(a) $\lim\limits_{x\to\infty} \dfrac{1}{x^2}$ **(b)** $\lim\limits_{x\to\infty} \dfrac{1}{\sqrt{x}}$ **(c)** $\lim\limits_{x\to\infty} e^{-x}$ **(d)** $\lim\limits_{x\to\infty} 2x$ **(e)** $\lim\limits_{x\to\infty} \sin x$

(11) Compute using algebra and the limit laws:

$$\lim_{x\to\infty} \frac{2x^2+1}{3x^2-x}$$

(12) Compute using algebra and the limit laws:

$$\lim_{x\to\infty} \frac{\sqrt{x^2+4}}{2x+5}$$

(13) Compute using algebra and the limit laws:

$$\lim_{x\to\infty} \sqrt{x^2+3} - x$$

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.
- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).
- Ask me about it after class.
- Come to my office hours: Mon 12:00 - 1:00, Wed 12:00 - 1:00 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.