Mth 31, Homework 10 on sections 4.7, 4.8, 4.9

Due by Wed, Nov 20.

Try these questions. Write all your working out and answers by hand on your own notepaper and hand them to me next week. Please use lots of space and as many pages as you want, so I can include corrections or comments. You do not need to write the questions, but it is very important that you show clearly any work you had to do to get your answers. Each question is worth 3 points.

Section 4.7 Optimization

(1) Find two numbers with sum 18 and product as large as possible using these steps:

- **(a)** Let x and y be the two numbers and $P = xy$ their product. Write the equation for "two numbers with sum 18".
- **(b)** Write P as a function of x only and compute $\frac{d}{dx}P$.
- **(c)** Find when $\frac{d}{dx}P = 0$. Is this a maximum? Write "The two numbers are .... and ...."

(2) Find the point on the line $y = 1 + 2x$ that is closest to the point $(5, 1)$.

(Hint: minimize the square of the distance between the points.)

(3) Find the coordinates of the two points on the ellipse $3x^2 + y^2 = 28$ that are furthest from the point $(6, 0)$.

(4) A farmer wants to enclose an area of 6 square miles with a rectangular fence. The fence should also divide the area in two with a line of fence running parallel to two sides. What is the shortest length of fence the farmer can use? Draw a diagram of this shortest fence.

(5) A plastic bucket in the shape of a circular cylinder with a base but no top is being designed to contain a volume of $8000\pi\ cm^3$. What should the dimensions be to minimize the amount of plastic.

(Hint: if the bucket has base of radius r and height h then the area of the base circle is πr^2, the area of the sides is $h \cdot 2\pi r$ and the volume is $h \cdot \pi r^2$.)

Section 4.8 Newton's method

(6) Use Newton's method to approximate a solution to $x^2 - 5 = 0$ as follows:

- **(a)** Starting with $x_1 = 2$, find the second approximation x_2.
- **(b)** Then find the third approximation x_3.
- **(c)** Compare your answers with the decimal of the exact solution.

(7) Use Newton's method to estimate $\sqrt[3]{24}$ using at least two steps, in other words start with an initial guess x_1 and find x_2 and x_3.

(Hint: that is the solution of $x^3 - 24 = 0$ so take $f(x) = x^3 - 24$.)

(8) Use Newton's method to estimate $\sqrt[4]{100}$ using at least two steps.

(9) Explain how you could use Newton's method to get an approximation for e, the base of the natural logarithm.

Section 4.9 Antiderivatives

(10) Find the most general antiderivative of

$$g(x) = 6x^5 - 10x^4 + 3x^3 + 1$$

and check that your answer is correct by differentiating it and getting $g(x)$ again.

(11) Find the most general antiderivative of: $x^{2/5} - \dfrac{2}{x^4} + x\sqrt{x} - \dfrac{8}{x}$

(12) What is the most general antiderivative of: $3^x + \pi + \sin x - 4\sec^2(x)$

(13) Find $f(x)$ if

$$f'(x) = e^x + \sinh x + \frac{2}{\sqrt{1-x^2}} \qquad \text{and} \qquad f(0) = 3$$

(14) Find $f(x)$ if

$$f''(x) = 20x^3 + 8 \qquad \text{and} \qquad f(0) = 5,\ f(1) = 0$$

(15) The position of an object at time t is $s(t)$. Its velocity is $v(t)$ and its acceleration is $a(t)$. Find the formula for $s(t)$ if

$$a(t) = 5\cos t - 3\sin t \qquad \text{and} \qquad s(0) = 0,\ v(0) = 5$$

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.
- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).
- Ask me about it after class.
- Come to my office hours: Mon 12:00 - 1:00, Wed 12:00 - 1:00 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.