MTH 31, Midterm extra Name (first, last): ....

If you got an F or a D on the midterm then you can work on this new exam to improve your grade, (F to D or D to C). Some of these questions will also be on the final exam.

Print out this exam (**it must be printed**). Follow the steps provided for each question, show all your work in the space provided, and make sure you get the correct answer if it is shown. Let me know if you have any questions or try the tutoring lab for help. Then bring this completed exam and your original midterm to my office hours and I will ask you a few questions to check you understand your solutions.

Q1. Numerically estimate:

$$\lim_{x\to\mathbf{0}^+} (3x)^x$$

Method: This is a right-sided limit so use $x = 0.01$ and $x = 0.001$ for example. See what numbers you get after substituting – what are they getting close to as x gets smaller?

Answer: 1

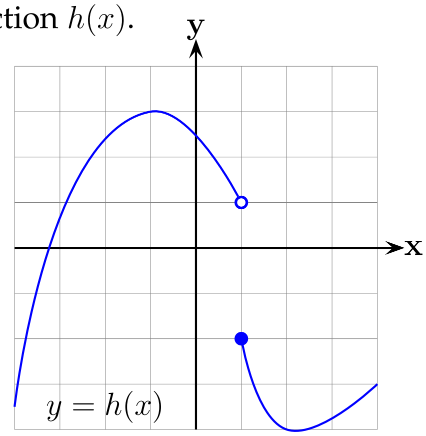
Q2. Compute this limit using algebra and the limit laws:

$$\lim_{x \to 4} \frac{\frac{3}{4} - \frac{3}{x}}{x - 4}$$

Method: Simplify the rational expression as follows. First combine $\frac{3}{4} - \frac{3}{x}$ on top using the LCD which is $4x$. Also, dividing by $x - 4$ is the same as multiplying by $\frac{1}{x-4}$. Cancel the $x - 4$ factors and now you can take the limit without getting $\frac{0}{0}$.

Answer: $\frac{3}{16}$

Q3. This is the graph of the function $h(x)$.



Find the following:

(a) $h(-2)$

(b) $h(1)$

(c) $\lim\limits_{x \to \mathbf{1}^-} h(x)$

(d) $\lim\limits_{x \to \mathbf{1}^+} h(x)$

(e) $\lim\limits_{x \to \mathbf{1}} h(x)$

Method: This question is asking for y values of points on the graph corresponding to the given x values. For the limit in part (c) the x values are approaching 1 from the left. From the right in (d). And from both sides in (e).

Q4. Let $f(x)$ be the function defined by

$$f(x) = \begin{cases} 1 - x & \text{if } x < 1 \\ x^2 - 1 & \text{if } x \geq 1. \end{cases}$$

(a) Sketch the graph of $f(x)$.

(b) Does $\lim_{x\to 1} f(x)$ exist?

(c) Explain if $f(x)$ is continuous or not at $x = 1$.

Method: The graph is a line to the left of $x = 1$ and part of a parabola to the right of $x = 1$. Part (b) is asking if the two parts meet up at $x = 1$ and what the common y value is if they do. (They do.) We have $f(x)$ continuous at $x = 1$ if $\lim_{x\to 1} f(x) = f(1)$. Is that true?

Q5. Let $f(x) = x^3 - 3x + 8$.

(a) Where is $f(x)$ continuous?

(b) Explain why there is at least one solution to the equation

$$x^3 - 3x + 8 = 0$$

in the interval $(-3, 1)$.

Method: Polynomials are continuous everywhere – write this in interval notation. Part (b) needs the Intermediate Value Theorem. Compute $f(-3)$ and $f(1)$ and check if the value 0 we want is between those two numbers. If it is then the theorem says that there must be a c in the interval $(-3, 1)$ with $f(c) = 0$. Then c is the solution we were after.

Q6. Compute using algebra and limit properties:

$$\lim_{x\to\infty} \frac{20x + 1}{9x^2 - x}$$

Method: The highest power of x on the bottom is x^2. Divide top and bottom by this:

$$\lim_{x\to\infty} \frac{20x + 1}{9x^2 - x} = \lim_{x\to\infty} \frac{\frac{20x}{x^2} + \frac{1}{x^2}}{\frac{9x^2}{x^2} - \frac{x}{x^2}}$$

Now simplify and see what happens as x gets bigger and bigger.

Answer: 0

Q7. For $g(x) = 5x^2 - 4x + 3$ use the limit definition of the derivative to find $g'(x)$.
Method: We are using

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}.$$

To find $g(x+h)$ just substitute $x+h$ for x in gs formula. Also use that $-g(x) = -(5x^2 - 4x + 3) = -5x^2 + 4x - 3$. Simplify, cancel h from top and bottom, and then let h go to 0.

Answer: $10x - 4$

Q8. Sketch a graph of a continuous function $f(x)$ if you know that:

$$\lim_{x \to -\infty} f(x) = 3, \quad f'(-2) = -1, \quad f'(0) = 0, \quad f'(3) = 1, \quad \lim_{x \to \infty} f(x) = 2$$

Method: This graph should have a horizontal asymptotes at $y = 3$ on the left and $y = 2$ on the right. Draw the curve so that it has the correct slopes at $x = -2, 0, 3$.

Q9. Find the equation of the tangent line to $y = x^4 + 3e^x - 2 + \sin x$ at the point $(0, 1)$.

Method: First find the derivative $\frac{dy}{dx}$. Evaluate this at $x = 0$ to get the slope of the tangent line. Then use the point slope formula $y - y_1 = m(x - x_1)$ and simplify to get the answer.

Answer: $y = 4x + 1$

Q10. Compute:

(a) $\dfrac{d}{dx}\left(4\sin x - 3\cos x + 2\right)$ To start, it equals $4\dfrac{d}{dx}\sin x - 3\dfrac{d}{dx}\cos x + \dfrac{d}{dx}2$

(b) $\dfrac{d}{dt}\left(e^t \cos t\right)$ Use the product rule.

(c) $\dfrac{d}{dx}\left(\dfrac{x^2+1}{x+2}\right)$ Use the quotient rule.

(d) $\dfrac{d}{d\theta}\left(-8\tan\theta\right)$ To start, it equals $-8\dfrac{d}{d\theta}\tan\theta$

Answer to (b): $e^t(\cos t + \sin t)$

Q11. Compute: (these all need the chain rule)

(a) $\dfrac{d}{dx}(1+x^3)^{14}$

(b) $\dfrac{d}{dx}3^{\sqrt{x}}$

(c) $\dfrac{d}{d\theta}\sin(\tan\theta)$

Answer to (a): $42x^2(1+x^3)^{13}$

Q12. If $xe^y = 3x - 2y$ then find $\dfrac{dy}{dx}$ using implicit differentiation.
Method: Take the derivative $\frac{d}{dx}$ on both sides and think of y as a function of x:

$$\frac{d}{dx}(xe^y) = 3\frac{d}{dx}x - 2\frac{d}{dx}y$$

We need the product rule on the left and also $\frac{d}{dx}e^y = e^y\frac{dy}{dx}$ by the chain rule. Finally solve for $\frac{dy}{dx}$.

Answer: $\dfrac{3 - e^y}{xe^y + 2}$