Review of <u>6.2</u> Factoring trinomials

Want to factor ax^2+bx+c → easier $a=1$
→ harder $a \neq 1$

Example 1.
Factor $x^2-2x-35$

Solution. We are in the easier case and don't need the ac-method here.

$x^2-2x-35$

$a=1$ $b=-2$ $c=-35$

Want $c=-35=(\ )(\ )$
$b=-2 \ =(\ )+(\ )$

Since $35=(5)(7)$ we have $-35=\underbrace{(-5)(7)}_{\text{wrong sum}}=\underbrace{(5)(-7)}_{\text{right sum}}$

Get $\boxed{(x+5)(x-7)}$.

Example 2.
Factor $3x^2+5x+2$

Solution. We are in the harder case and need the ac-method.

This method says to look at
$ac=6 \ =(\ )(\ )$
$b=5 \ =(\ )+(\ )$

The numbers we need to fill in are 2 and 3.

Next step is to split the middle term bx with these numbers:

$$3x^2 + 5x + 2$$

$$= 3x^2 + 2x + 3x + 2$$

Then factor this by grouping

$$= x(3x+2) + 1(3x+2)$$

$$= (x+1)(3x+2).$$

FOIL to check, but this is our answer

$$3x^2+5x+2 = \boxed{(x+1)(3x+2)}.$$

Note that the same steps also factor

$$3x^2+5xy+2y^2$$

and we get

$$3x^2+5xy+2y^2$$

$$= 3x^2 + 2xy + 3xy + 2y^2$$

$$= x(3x+2y) + y(3x+2y)$$

$$= \underline{(x+y)(3x+2y)}.$$

6.3 Factor special products

The first special product is a "perfect square trinomial" and it gives us a short cut.

Example 1.
Factor $9x^2 + 42x + 49$

Solution. Well we could use the ac-method here. There is a quicker way here though.

Notice that $9x^2 = (3x)^2$ and $49 = (7)^2$

also $42x = 2(3x)(7)$.

This is what you get from multiplying out

$$(3x+7)^2 = (3x+7)(3x+7)$$

$$\begin{array}{l} \qquad\quad \text{F} \qquad\qquad \text{O} \qquad\quad\ \text{I} \qquad\quad \text{L} \\ = (3x)(3x) + (3x)(7) + (3x)7 + (7)^2 \end{array}$$

$$= 9x^2 + 42x + 49.$$

So the answer is $\boxed{(3x+7)^2}$

The special product we are using is in general

$$(a+b)^2 = a^2 + 2ab + b^2$$

and we try to match it to what we're factoring.

We can also make the middle term negative

$$(a-b)^2 = a^2 - 2ab + b^2.$$

Example 2.
Factor $81x^2 + 36x + 4$.

Solution. $81x^2$ is a square $= (9x)^2$
and 4 is a square $= (2)^2$

Do we have $a^2 + 2ab + b^2$?

$a = 9x$, $b = 2$, $2ab = 2(9x)(2) = 36x$

Yes, that is the middle term

So $81x^2 + 36x + 4 = \boxed{(9x+2)^2}$.

Example 3.
Factor $16x^2 - 40xy + 25y^2$

Solution. We recognize $16x^2 = (4x)^2$
and $25y^2 = (5y)^2$

Also $40xy = 2(4x)(5y)$

but we need a minus sign:

$$\boxed{(4x - 5y)^2}$$ Check by Foiling.

Answer.

The second special product is called a "difference of squares".

It is based on this: $\overbrace{(a+b)}^{\text{sum}}\overbrace{(a-b)}^{\text{difference}}$

$$= a^2 - ab + ab - b^2$$

$$= \underbrace{a^2 - b^2}_{\text{difference of squares}}$$

so that

$$a^2 - b^2 = (a+b)(a-b).$$

Example 1.
Factor $x^2 - 16$.

Solution. We are looking for $(\ )^2 - (\ )^2$

and have that here with $x^2 - 16 = (\underset{a}{x})^2 - (\underset{b}{4})^2$

and the factors are $(x+4)(x-4)$.

Example 2.
Factor $121x^2 - 1$

Solution. This also matches our template

$$121x^2 - 1 = (11x)^2 - (1)^2$$

and so $121x^2 - 1 = (11x+1)(11x-1)$.

Example 3.
Factor $8y^2 - 200x^2$.

Solution. This does not match our template. But it can be simplified by first factoring out the GCF of the two terms:

$$8y^2 - 200x^2 = 8(y^2 - 25x^2)$$

and $$y^2 - 25x^2 = (y)^2 - (5x)^2 = (y+5x)(y-5x)$$

Altogether
$$8y^2 - 200x^2 = \boxed{8(y+5x)(y-5x)}.$$
Answer.

How about factoring a sum of squares?

$$x^2 + 16 = (\quad)(\quad)\ ?$$

This is not possible, so we just say x^2+16 does not factor.

a^2+b^2 does not factor.