6.4 General strategy for factoring.

Factoring is an important skill we will need for simplifying pieces of algebra and for solving equations.

How do we know which of the methods in this chapter to use?

- Always look for a GCF to factor out first.
- For a binomial (2 terms) see if it is a difference of squares.
- For a trinomial (3 terms) we have
  - easy case eg. $x^2+5x+6=(x+2)(x+3)$
  - perfect square trinomial eg. $4x^2+12x+9=(2x+3)^2$
  - harder case eg. $3x^2+5x+2=(x+1)(3x+2)$ needing the ac-method.
- For 4 terms we can try factoring by grouping.

If you write down a random polynomial it might not have any factors, for example

$$x^2+x+1.$$

Example 1.
Factor completely $20xy - 30x - 4y + 6$

Solution. All the coefficients are even, so 2 is a common factor. The GCF is 2:

$$20xy - 30x - 4y + 6 = 2(10xy - 15x - 2y + 3)$$

Factor by grouping

$$10xy - 15x - 2y + 3$$

$$= 5x(2y-3) - 1(2y-3)$$

$$= (5x-1)(2y-3)$$

Altogether the factors are $2(5x-1)(2y-3)$.

Example 2.
Factor completely: $45c^2 - 30cd + 5d^2$

Solution. The GCF is 5:

$$45c^2 - 30cd + 5d^2 = 5(9c^2 - 6cd + d^2)$$

To factor the trinomial we see that we're not in the easy case. The ac-method is possible but first check if its in the form of a perfect square trinomial

$$(\ )^2 \pm 2(\ )(\ ) + (\ )^2$$

It is $9c^2 = (3c)^2 \qquad d^2 = (d)^2$

and $-6cd = -2(3c)(d)$

so $9c^2 - 6cd + d^2 = (3c-d)^2$

Altogether the factors are $5(3c-d)^2$.

(In fact we could have used the easy case here by switching the order

$$9c^2 - 6cd + d^2 = d^2 - 6cd + 9c^2$$

$$= (d \quad)(d \quad) \qquad 9 = (\;)(\;)$$

$$-6 = (\;)+(\;)$$

$$= (d-3c)(d-3c).$$

)

Example 3.

Factor completely $3xk^4 - 48x$

Solution. Factor out the GCF first

$$3xk^4 - 48x = 3x(k^4 - 16)$$

Next we recognize $k^4 - 16 = (k^2)^2 - (4)^2$

so $k^4 - 16 = (\underset{\substack{\uparrow \\ \text{does not} \\ \text{factor}}}{k^2+4})(\underset{\substack{\uparrow \\ \text{does} \\ \text{factor}}}{k^2-4})$ $= (k+2)(k-2)$

Answer: $3x(k^2+4)(k+2)(k-2)$.

Example 4.

Factor completely $3x^3-8x^2+4x$

Solution. GCF $= x$ this time so

$$3x^3-8x^2+4x = x\left(\underbrace{3x^2}_{\substack{\text{not}\\ \text{a square}}}-8x+4\right)$$

Our only option is the ac-method here

$$\underset{\overset{\nearrow}{a=3}}{3x^2}\quad \underset{\overset{\uparrow}{b=-8}}{-8x}\quad \underset{\overset{\uparrow}{c=4}}{+4}$$

$ac = 12 = (\ \ )(\ \ )$

$b = -8 = (\ \ ) + (\ \ )$

possible products	$(1)(12)$	sum 13	
	$(-1)(-12)$	-13	
	$(2)(6)$	8	
	$(-2)(-6)$	-8	←
	$(3)(4)$	7	
	$(-3)(-4)$	-7	

Get $$3x^2 \underset{\swarrow\ \ \searrow}{-8x} + 4$$

$$= 3x^2-2x-6x+4$$

$$= x(3x-2)-2(3x-2)$$

$$= (x-2)(3x-2)$$

Answer: factors are $\boxed{x(x-2)(3x-2)}$