Mth 33, Homework 4 on sections 13.1 – 14.2

Due by Wed, Mar 4.

Please use lots of space and explain your answers, showing clearly any work you had to do. Each question is worth 3 points.

Sections 13.1, 13.2 Vector Functions, Space Curves and Derivatives

(1) Sketch the space curve obtained from the vector function

$$\mathbf{r}(t) = \langle t\cos t, t\sin t, t\rangle, \qquad t \geqslant 0.$$

(2) For the same $\mathbf{r}(t)$ in the last question, find

- **(a)** $\mathbf{r}'(t)$
- **(b)** $\mathbf{r}(0)$
- **(c)** $\mathbf{r}'(0)$
- **(d)** $\mathbf{T}(0)$, which is the unit tangent vector at $t = 0$.

(3) Find $\mathbf{T}(1)$ when

$$\mathbf{r}(t) = \langle t^3 + 1, 3t - 5, 4/t\rangle.$$

(4) Let

$$\mathbf{r}(t) = e^{3t}\mathbf{i} + (-1 + \sin(2t))\mathbf{j} + \ln(t+1)\mathbf{k}$$

and find the parametric equations of the tangent line to this space curve when $t = 0$

(5) For the vector functions

$$\mathbf{u}(t) = \langle 1, t, t^3\rangle, \qquad \mathbf{v}(t) = \langle t^2, 3, t^5\rangle$$

use the product rule to compute

$$\frac{d}{dt}\left(\mathbf{u}(t)\cdot\mathbf{v}(t)\right)$$

and give your answer as a simplified function of t.

Section 13.3 Arc Length and Curvature

(6) Use the arc length formula

$$L = \int_a^b |\mathbf{r}'(t)|\,dt$$

to find the length of the curve

$$\mathbf{r}(t) = \langle 2t, t^2, \tfrac{1}{3}t^3\rangle, \qquad 0 \leqslant t \leqslant 1.$$

(7) For the curve given by $\mathbf{r}(t) = \langle 3t, 2t+1, 6t-1 \rangle$, do the following:

- **(a)** Find the arc length function $s(t)$ giving the length from $(0, 1, -1)$ to $\mathbf{r}(t)$.
- **(b)** Reparametrize the curve with respect to this arc length, writing $\mathbf{r}$ in terms of s.
- **(c)** If you move 14 units along the curve from the point $(0, 1, -1)$, where are you now?

(8) For the curve given by $\mathbf{r}(t) = \langle 3\sin t, t, 3\cos t \rangle$, do the following:

- **(a)** Find its unit tangent vector $\mathbf{T}(t)$.
- **(b)** Find its unit normal vector $\mathbf{N}(t)$.
- **(c)** Find its curvature $\kappa(t)$.

Formulas needed: $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$, $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}$, $\kappa(t) = \left| \dfrac{\mathbf{T}'(t)}{\mathbf{r}'(t)} \right|$.

(9) For the curve given by $\mathbf{r}(t) = \langle t, t^2, e^t \rangle$, find its curvature $\kappa(t)$ using the alternate formula

$$\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}.$$

(10) For a usual function $y = f(x)$, the curvature of its graph at x is

$$\kappa(x) = \frac{|f''(x)|}{(1 + (f'(x))^2)^{3/2}}.$$

Let $f(x) = x^3$.

- **(a)** Find its curvature $\kappa(x)$.
- **(b)** Find $\kappa(1)$.
- **(c)** A circle of what radius has curvature $\kappa(1)$?

Section 14.1 Functions of Several Variables

(11) Let $f(x, y) = \dfrac{\sqrt{x+y}}{y}$ be a function of two variables.

- **(a)** Evaluate $f(1, 3)$.
- **(b)** Find and sketch the domain of f.

(12) Find and sketch the domain of $g(x, y) = \ln(4 - x^2 - y^2)$.

Section 14.2 Limits and Continuity

(13) Let $f(x, y) = \dfrac{x \sin(2y)}{y \sin(x)}$. Compute $f(0.01, 0.01)$ with your calculator (in radians mode) and guess what $\lim_{(x,y)\to(0,0)} f(x, y)$ could be.

(14) Let $g(x, y) = \dfrac{xy}{3x^2 + y^2}$ Show that the limit as $(x, y) \to (0, 0)$ of $g(x, y)$ does not exist. Do this by writing $y = mx$ and showing that you get different limits for different ms.

(15) For the same $g(x, y)$ as in the last question,

- **(a)** What property of rational functions says that the limit as $(x, y) \to (1, 2)$ of $g(x, y)$ does exist?
- **(b)** Find this limit.

If you are stuck on a question:

- Ask me about it after class.
- Come to my office hours: Mon 4:30 - 5:30, Wed 4:30 - 5:30 in CP 317.
- Go to the Math Tutorial Lab in person in CP 303 or online.