Mth 30, Homework 12 on sections 7.1, 7.2, 7.5

Due by Wed, May 6.

Please use lots of space and explain your answers, showing clearly any work you had to do. Each question is worth 3 points.

Section 7.1 Trigonometric Identities

(1) Fill in the following blanks describing how each step works. For example it could be "by the Pythagorean identity", "adding fractions", "by definition" or "distributing".

$$\begin{aligned}
\frac{\cot x + \tan x}{\sec x} &= \frac{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}}{\frac{1}{\cos x}} && \text{_____}\\
&= \cos x \left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} \right) && \text{_____}\\
&= \frac{\cos^2 x + \sin^2 x}{\sin x} && \text{_____}\\
&= \frac{1}{\sin x} && \text{_____}\\
&= \csc x. && \text{_____}
\end{aligned}$$

The above steps have verified the identity $\dfrac{\cot x + \tan x}{\sec x} = \csc x$.

(2) Simplify

$$\frac{3\sin^2 x + 4\cos^2 x - 3}{\cos x}$$

to a single trigonometric function. Go step by step as in question 1 and don't forget to write the equality symbol "=" at each step, to show what is equal.

(3) Verify the identity:

$$\sin(-x)\tan x + \sec(-x) = \cos x$$

(Remember that $\cos$ and $\sec$ are even functions and the other trig functions are odd.)

(4) Verify the identity:

$$\cot^2\theta - \csc^2\theta = -1$$

(5) Verify the identity:

$$2 = \frac{(\cos\theta + 1)^2 + (\sin\theta + 1)^2 - 3}{\cos\theta + \sin\theta}$$

Section 7.2 Sum and Difference Identities

Use these sum identities in these three questions:

$$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta$$
$$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta$$

(6) Find the exact value of: $\cos(105^\circ)$

(Hint: Write 105° as a sum of special angles.)

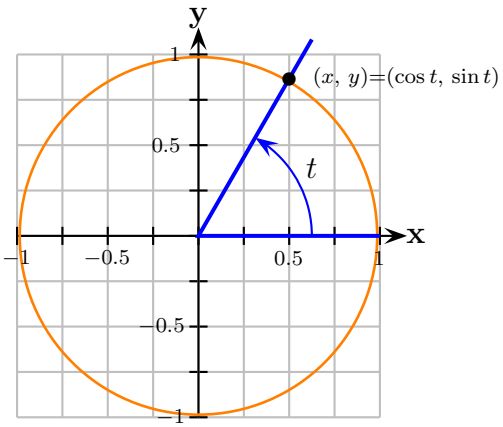
(7) Write $\sin(x-\pi/4)$ in terms of $\sin x$ and $\cos x$.

(8) Verify the identity:

$$\frac{\sin(x+y)}{\sin x\cdot\sin y} = \cot x + \cot y$$

Section 7.5 Solving Trigonometric Equations

The unit circle, as shown, can be helpful for solving trigonometric equations.



(9) Find all solutions to: $\sin t = 1$

(Hint: There are infinitely many – find radian angles that have $y = 1$ on the unit circle above. The answer is "$t = ?? + 2\pi k$ for all integers k".)

(10) Solve $1+\cos t = 0$ exactly for t in $[0, 6\pi)$.

(Use the unit circle diagram to find the three solutions.)

(11) Solve $1+3\sin t = \sin t$ exactly for t in $[0, 2\pi)$ using these steps:

- **(a)** Simplify the equations to look like "$\sin t =$ a number".
- **(b)** Find the special angle that has its sine equal to the positive part of that number.
- **(c)** We are looking for angles t that have this special angle as reference angle, and are in the correct quadrant to give the right sign.

(d) Write the two solutions for t.

(12) Use the same steps to solve $10\cos(\theta) + 2\sqrt{2} = 6\cos(\theta)$ exactly for θ in $[0, 2\pi)$.

(13) Solve $\sin^2 t = 3/4$ exactly for t in $[0, 2\pi)$. (Find all four solutions.)

If you're stuck on a question:

- Ask me about it after class.
- Come to my office hours: Mon 4:30 - 5:30, Wed 4:30 - 5:30 in CP 317.
- Go to the Math Tutorial Lab in person in CP 303 or online.