Mth 30, Homework 12 on sections 7.1, 7.2, 7.5

Due by Mon, May 13.

Please use lots of space and explain your answers, showing clearly any work you had to do. Each question is worth 3 points.

Questions 5, 6, 7 below involve the sum identities:

$$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta$$
$$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta$$

(1) Simplify

$$\frac{3\sin^2 x + 4\cos^2 x - 3}{\cos x}$$

to a single trigonometric function. Explain your steps.

(2) Fill in the following blanks describing how each step works. For example it could be "using the Pythagorean identity" or "adding fractions" or "distributing".

$$\begin{aligned}
\frac{\cot x + \tan x}{\sec x} &= \frac{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}}{\frac{1}{\cos x}} \qquad \text{_____} \\
&= \cos x\left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right) \qquad \text{_____} \\
&= \frac{\cos^2 x + \sin^2 x}{\sin x} \qquad \text{_____} \\
&= \frac{1}{\sin x} \qquad \text{_____} \\
&= \csc x. \qquad \text{_____}
\end{aligned}$$

The above steps have verified the identity $\dfrac{\cot x + \tan x}{\sec x} = \csc x$.

(3) Use the same format as the last question, and similar steps, to verify the identity

$$\sin(-x)\tan x + \sec(-x) = \cos x.$$

(Remember that $\cos$ and $\sec$ are even functions and the other trig functions are odd.)

(4) Verify the identity

$$\cot^2\theta - \csc^2\theta = -1.$$

(5) Find the exact value of: $\cos(105^\circ)$

(Hint: Write 105° as a sum of special angles.)

(6) Write $\sin(x - \pi/4)$ in terms of $\sin x$ and $\cos x$.

(7) Verify the identity:

$$\frac{\sin(x+y)}{\sin x \cdot \sin y} = \cot x + \cot y$$

(8) Find all solutions to: $\sin x = 1$

(Hint: There are infinitely many. The answer is "$x = ?? + 2\pi k$ for all integers k".)

(9) Solve $3\sin(x) = 1 + \sin(x)$ exactly for x in $[0, 2\pi)$.

(Hint: There are two solutions. On the unit circle you will be looking for two points with y coordinate $1/2$ and see which angles they correspond to.)

(10) Solve $2\cos(2x) = 1$ exactly for x in $[0, 2\pi)$. (Now four solutions.)

(11) Solve exactly for θ in $[0, 2\pi)$:

$$\sin^2\theta = 3/4$$

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.
- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).
- Ask me about it after class.
- Come to my office hours: Mon 12:00 - 1:00, Wed 12:00 - 1:00 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.