Math 34, Exam 1 on Chapters 1, 2. Version 2 Answers

For the questions that you got wrong or lost points on, try to do them again using your notes and the book. You should be getting the answers given below. Similar questions to these will be on the final exam.

Q1. For the following differential equation,

$$\frac{d^2y}{dt^2} + y\frac{dy}{dt} + 7y = \sin(t)$$

(a) Give its order: **its order is** 2

(b) Say if it is linear or nonlinear: **it is nonlinear**

Q2. Decide if $y = 5e^{-t}$ a solution to the equation:

$$y'' - 2y' - 3y = 0$$

It is a solution.

Q3. Find the general solution of:

$$y' = 4y - 2e^t$$

The general solution is

$$y = \frac{2}{3}e^t + Ce^{4t}$$

Q4. Give the solution to the ODE in question 3 that has the initial value: $y(0) = 4$
This solution is

$$y = \frac{2}{3}e^t + \frac{10}{3}e^{4t}$$

Q5. Solve the separable equation:

$$\sin(x) + 2 + 3y^2 \frac{dy}{dx} = 0$$

The general solution is

$$-\cos(x) + 2x + y^3 = C$$

Q6. Draw the direction field for the ODE

$$\frac{dy}{dt} = 2 - y$$

and use this picture to explain how y behaves as $t \to \infty$.

With the direction field you should see that $y \to 2$ as $t \to \infty$.

Q7. A tank contains 500 gallons of water. Suppose there are $Q(t)$ lbs of salt in the tank at time t, measured in minutes, with the initial value $Q(0) = 0$. Water containing 0.2 lbs of salt per gallon enters the tank at a rate of 5 gal/min and the same amount of water leaves the tank after mixing.

(a) Model this situation with a differential equation.

(b) Solve this equation.

(c) How much salt is in the tank after 100 minutes?

The differential equation is

$$\frac{dQ}{dt} + \frac{Q}{100} = 1$$

with solution

$$Q = 100 - 100e^{-t/100}$$

and $Q(100) \approx 63.21$ **lbs of salt.**

Q8. Remember that the differential equation

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

is exact if $\dfrac{\partial}{\partial y}M(x,y) = \dfrac{\partial}{\partial x}N(x,y)$. For the equation

$$2xy^2 + 2y + (2x^2y + 2x + 3)\frac{dy}{dx} = 0$$

(a) Decide if it is exact or not.

(b) If it is exact, find the general solution using $M = \dfrac{\partial}{\partial x}f(x,y)$ and $N = \dfrac{\partial}{\partial y}f(x,y)$.

The equation is exact with solution

$$x^2y^2 + 2xy + 3y = C$$

Q9. Consider the ODE

$$\frac{dy}{dt} + y\sin(t) = \frac{1}{t-3}$$

with initial value $y(0) = 2$. Explain for which values of t a solution $y(t)$ must exist. (Do not solve the equation.)

Solutions exist for all $t < 3$

Q10. For the autonomous equation

$$\frac{dy}{dt} = (y-2)y(y+1)$$

(a) First draw the graph of $f(y) = (y-2)y(y+1)$

(b) Find the critical equilibrium points and classify them as stable or unstable.

(c) Draw the phase line (y axis with arrows) and sketch some graphs of solutions to the autonomous equation in the ty-plane.

The equilibrium points are at $y = -1$ **(unstable),** $y = 0$ **(stable) and** $y = 2$ **(unstable). Draw the phase line and sketch solutions.**