Math 35, Homework 8 on Sections 16.6, 16.7, 16.8
due Wed, May 7 at the start of class.

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(1) Let T be the triangular region with vertices $(1, 0, 0)$, $(0, 2, 0)$ and $(0, 0, 2)$. Compute the surface integral of xy over T. In other words, find:

$$\iint_T xy\, dS$$

(2) Let S be the surface given by

$$\mathbf{r}(u, v) = u^2\mathbf{i} + u \sin v\mathbf{j} + u \cos v\mathbf{k} \quad \text{where} \quad 0 \leqslant u \leqslant 1,\ 0 \leqslant v \leqslant \pi/2.$$

Find:

$$\iint_S yz\, dS$$

(3) Let

$$\mathbf{F}(x, y, z) = x\mathbf{i} + x^2\mathbf{j} - y\mathbf{k}$$

and let T be the part of the paraboloid $z = x^2 + 3y^2$ that lies above the square $-1 \leqslant x \leqslant 1$, $-1 \leqslant y \leqslant 1$. Evaluate:

$$\iint_T \mathbf{F} \cdot d\mathbf{S}$$

(4) Let

$$\mathbf{F}(x, y, z) = xy\mathbf{i} + x\mathbf{j} + z\mathbf{k}$$

and let H be the hemisphere $x^2 + y^2 + z^2 = 25$, $z \geqslant 0$ oriented away from the origin. Evaluate the flux of $\mathbf{F}$ across H:

$$\iint_H \mathbf{F} \cdot d\mathbf{S}$$

(5) Let

$$\mathbf{F}(x, y, z) = 2y \cos z\mathbf{i} + e^x \sin z\mathbf{j} + xe^y\mathbf{k}$$

and let S be the hemisphere $x^2 + y^2 + z^2 = 9$, $z \geqslant 0$ oriented upward. Use Stokes' Theorem to evaluate:

$$\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$$

(6) Use Stokes' Theorem to evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where

$$\mathbf{F}(x, y, z) = e^{-x}\mathbf{i} + e^x\mathbf{j} + e^z\mathbf{k}$$

and C is the boundary of the part of the plane $2x + y + 2z = 2$ in the first octant, oriented counterclockwise as viewed from above.

(7) Verify that the Divergence Theorem is true for the vector field

$$\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

and the region E given by the unit ball $x^2 + y^2 + z^2 \leqslant 1$ by computing both sides.

(8) Use the Divergence Theorem to calculate the flux of $\mathbf{F}$ across S where S is the surface of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + 2y + z = 2$ and

$$\mathbf{F}(x, y, z) = x^2y\mathbf{i} + xy^2\mathbf{j} + 2xyz\mathbf{k}.$$