Math 35, Homework 7 on Sections 16.4, 16.5, 16.6
due Wed, Apr 23 at the start of class.

---

(1) Evaluate the line integral

$$\int_C (x-y)\,dx + (x+y)\,dy$$

directly, where C is the circle $x^2+y^2=4$, oriented positively.

(2) Evaluate the integral in the last question using Green's Theorem.

(3) Use Green's Theorem to evaluate

$$\int_C \mathbf{F}\cdot d\mathbf{r}$$

where

$$\mathbf{F}(x,y) = y^2\cos x\mathbf{i} + (x^2+2y\sin x)\mathbf{j}$$

and C is the triangle from $(0,0)$ to $(2,6)$ to $(2,0)$ to $(0,0)$.

(4) Find the area of the triangle D with vertices $(0,0)$, $(1,3)$ and $(4,4)$ using Green's Theorem. For example you could use the area formula:

$$A(D) = \oint_{\partial D} x\,dy$$

(5) Define the vector field

$$\mathbf{F}(x,y,z) = e^x\mathbf{i} + e^{xy}\mathbf{j} + e^{xyz}\mathbf{k}$$

and compute

(a) its divergence: $\nabla\cdot\mathbf{F}$

(b) its curl: $\nabla\times\mathbf{F}$.

(6) Let

$$\mathbf{G}(x,y,z) = xy^2z^2\mathbf{i} + x^2yz^2\mathbf{j} + 2x^2y^2z\mathbf{k}$$

Show $\mathbf{G}$ is not conservative by finding $\nabla\times\mathbf{G}$.

(7) Let $\mathbf{F}$ and $\mathbf{G}$ be two differentiable vector fields with components

$$\mathbf{F} = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}, \qquad \mathbf{G} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}.$$

Prove the identity

$$\nabla\cdot(\mathbf{F}\times\mathbf{G}) = \mathbf{G}\cdot(\nabla\times\mathbf{F}) - \mathbf{F}\cdot(\nabla\times\mathbf{G}).$$

(8) Let

$$\mathbf{r}(u,v) = \langle u+v, u^2-v, u+v^2\rangle$$

be a surface parameterized by $(u,v)\in\mathbb{R}^2$. Are either of the points $P(3,-1,5)$ and $Q(-1,3,4)$ on this surface?

(9) Let S be the surface parameterized by

$$\mathbf{r}(u, v) = 3\cos u\mathbf{i} + 3\sin u\mathbf{j} + v\mathbf{k} \quad \text{where} \quad 0 \leqslant u \leqslant \pi/2,\ 0 \leqslant v \leqslant 2.$$

Sketch S.

(10) Find the surface area of S from the previous question with the formula

$$A(S) = \iint_D |\mathbf{r}_u \times \mathbf{r}_v| \, dA.$$