Math 35, Homework 6 on Sections 16.1, 16.2, 16.3
due Wed, Apr 2 at the start of class.

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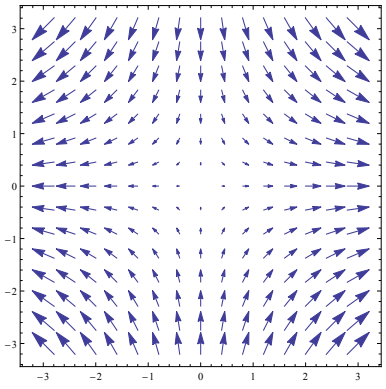
(1) The vector field shown here



is given by which of these following expressions? Explain.

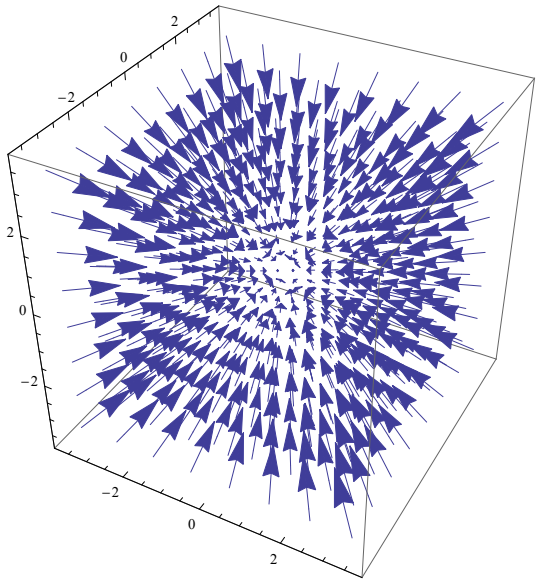
(i) $x\mathbf{i} + y\mathbf{j}$

(ii) $x\mathbf{i} - y\mathbf{j}$

(iii) $y\mathbf{i} + x\mathbf{j}$

(2) Find the gradient vector field ∇f of $f(x, y) = \sqrt{x^2 + y^2}$. Determine the length and direction of each vector in this vector field and then sketch the vector field.

(3) This vector field



is given by which of these following expressions? Explain.

(i) $x\mathbf{i} + y\mathbf{j} + 1\mathbf{k}$

(ii) $-1\mathbf{i} - 1\mathbf{j} - 1\mathbf{k}$

(iii) $-x\mathbf{i} - y\mathbf{j} - z\mathbf{k}$

(4) Let C be the curve parametrized by $(x(t), y(t)) = (t^2, 2t)$ for $0 \leqslant t \leqslant 1$. Find the line integral:

$$\int_C xy\, ds$$

(5) Suppose C consists of the line segments from $(0, 0, 0)$ to $(1, 2, -1)$ and from $(1, 2, -1)$ to $(3, 2, 0)$. Evaluate:

$$\int_C x^2\, dx + y^2\, dy + z^2\, dz$$

(6) A thin wire has the shape of the first-quadrant part of the circle with center at the origin and radius a. If the linear density function is $\rho(x, y) = xy$, find the mass and center of mass of the wire.

(7) Determine whether or not the vector field

$$\mathbf{F}(x, y) = (3x^2 - 2y^2)\mathbf{i} + (4xy + 3)\mathbf{j}$$

is conservative. If it is find an f so that $\nabla f = \mathbf{F}$.

(8) Determine whether or not the vector field

$$\mathbf{F}(x, y) = e^x \sin y\mathbf{i} + e^x \cos y\mathbf{j}$$

is conservative. If it is find an f so that $\nabla f = \mathbf{F}$.

(9) Let $\mathbf{F}(x, y) = x^2\mathbf{i} + y^2\mathbf{j}$ be a vector field. Find f so that $\nabla f = \mathbf{F}$. Then, just by using the fundamental theorem for line integrals, evaluate the integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

for C the arc of the parabola $y = 2x^2$ from $(-1, 2)$ to $(2, 8)$.