6.5 Linear equations (continued)

The basic linear equation we want to solve looks like

$$ax + b = c$$

for numbers a, b, c with $a \neq 0$.

For example, with $a = 8$, $b = -3$, $c = 5$ this is

$$8x - 3 = 5.$$

As we saw last time, there are two steps to solve it. First, in step (A) add $-b$ on both sides (in other words subtract b). Secondly, in step (B) divide both sides by a:

To solve $ax + b = c$

(A) subtract b from both sides
then (B) divide both sides by a.

In our example

$$\begin{array}{rll} & 8x - 3 = 5 & \\ (A) & \quad +3 \quad +3 & \\ \hline & 8x \quad\quad = 8 & \\ (B) & \frac{8x}{8} \quad\quad = \frac{8}{8} & \end{array}$$

Answer: $x = 1$ is the solution.

Example ② Solve $-4x + 6 = 1$

Solution: $-4x + 6 = 1$ $\quad (a=-4,\ b=6,\ c=1)$

Ⓐ $\quad -6 \quad -6$

$-4x \quad = -5$

Then divide both sides by -4

Ⓑ $\quad \frac{-4x}{-4} = \frac{-5}{-4}$

and $x = \frac{5}{4}$ is the solution.

- See p 183.

Example ③ Solve $2x + 7x + 1 = -3 - 2$

Solution: We need an extra first step here to simplify each side a little. On the right we have $-3-2=-5$ and on the left $2x+7x=9x$. So the simplified equation is

$$9x + 1 = -5$$

and we continue with Ⓐ, Ⓑ:

$$9x + 1 = -5$$
$$-1 \quad -1$$
$$\frac{9x}{9} = \frac{-6}{9}$$

$x = -\frac{6}{9} = -\frac{2}{3}$ Ans. $\boxed{x = -\frac{2}{3}}$

Example (4) Solve $\frac{2}{3}x - 1 = 5$

Solution: Step (A) has

$$\begin{array}{r} \frac{2}{3}x - 1 = 5 \\ +1 \quad +1 \\ \hline \frac{2}{3}x \quad = 6 \end{array}$$

for step (B) we want to divide both sides by $\frac{2}{3}$. It is easier to multiply both sides by the reciprocal of $\frac{2}{3}$ which is $\frac{3}{2}$:

$$\frac{3}{2} \cdot \frac{2}{3}x = \frac{3}{2} \cdot 6$$

$$x = \frac{3}{2} \cdot \frac{6}{1} = 9 \qquad \text{Ans. } \boxed{x = 9}$$

More complicated linear equations.

We could have xs on both sides of the equation. In that case, subtract one of the linear terms from both sides so that you are left with xs on only one side.

Example (5) Solve $8x - 3 = 3x + 12$

Solution: The best first step here is to subtract $3x$ from both sides.

$$\begin{array}{r} 8x - 3 = 3x + 12 \\ -3x \qquad -3x \\ \hline 5x - 3 = \quad 12 \end{array}$$

we used $8x - 3x = 5x$ and $3x - 3x = 0x = 0$

then

$$5x - 3 = 12$$

(A) $+3 \quad +3$

$$5x = 15$$

(B) $\frac{5x}{5} = \frac{15}{5}$

$$x = 3.$$

Let's check our solution works in the original equation $8x - 3 = 3x + 12$

$$8(3) - 3 = 3(3) + 12 \ ?$$
$$24 - 3 = 9 + 12$$
$$21 = 21 \ \checkmark$$

Answer $\boxed{x = 3}$.

- See example p 184.

When solving equations the main ideas are

- At the start, simplify each side as much as possible.
- After that, each step should do the same thing to both sides.
- You want x on just one side in the end, equal to a number.
- Check your solution works in the original equation.

Example (6) Solve $5(x+2) = x+10$

Solution: We need to remember the distribution rule here $5(x+2) = 5x + 5\cdot 2$
$= 5x + 10$

and we get $5x + 10 = x + 10$

Next we want Xs on only one side so

$$\begin{array}{rrr} 5x & +10 & = x+10 \\ -x & & -x \\ \hline 4x & +10 & = 10 \end{array}$$

Next (A) $$\begin{array}{rrr} & -10 & -10 \\ \hline 4x & & = 0 \end{array}$$

(B) $$\frac{4x}{4} = \frac{0}{4}$$

$$x = 0$$

Answer $\boxed{x=0}$.

Example (7) Solve $-3(x-4)+x = 5$

Solution: Distribute first to find

$$-3x + 12 + x = 5$$

Next simplify the left side using $-3x + x$ so that

$$-2x + 12 = 5$$

$= -3x + 1x$
$= (-3+1)x$
$= -2x$

Subtract 12 from both sides

$$-2x + 12 = 5$$
$$-12 \quad -12$$
$$-2x = -7$$

divide both sides by -2

$$\frac{-2x}{-2} = \frac{-7}{-2}$$

and $x = \frac{7}{2}$ Answer $\boxed{x = \frac{7}{2}}$

Can check it solves $-3(x-4)+x=5$

$$-3\left(\left(\tfrac{7}{2}\right)-4\right)+\left(\tfrac{7}{2}\right)=5\ ?$$

$$\tfrac{7}{2} - \tfrac{8}{2} = -\tfrac{1}{2}$$

$$-\tfrac{3}{1}\left(-\tfrac{1}{2}\right) + \tfrac{7}{2} = 5\ ?$$

$$\tfrac{3}{2} + \tfrac{7}{2} = 5$$

$$\tfrac{10}{2}$$

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