6.4 Linear terms

A simple situation in algebra is when we have a number multiplied by a variable.
For example

$$4x$$

or

$$13y.$$

Definition: A linear term is a number times a variable.

More examples of linear terms:

- $2w$ number is 2, variable is w
- $\frac{4}{5}x$ number is $\frac{4}{5}$, variable is x
- $\frac{x}{3}$ number is $\frac{1}{3}$, variable is x
- $-6y$ number is -6, variable is y
- y number is 1, variable is y
- $-x$ number is -1, variable is x.

Adding linear terms

Remember the distribution rule of arithmetic

$$a(b+c) = ab + ac$$

$$(a+b)c = ac + bc$$

Using the second one we see for example

$$\underbrace{(2+3)}_{5}x = 2x + 3x$$

So $2x + 3x = (2+3)x = 5x$

This makes sense – if $x = 10$ it is saying

$$2 \cdot 10 + 3 \cdot 10 = 5 \cdot 10 \qquad (20 + 30 = 50 \checkmark)$$

So we can simplify sums of linear terms if they have the <u>same</u> variable.

<u>Example</u> 1 Simplify $12y + 19y$

Solution: $12y + 19y = (12+19)y = \boxed{31y}$.

<u>Example</u> 2 Simplify $3x - 10x$

Solution: $3x - 10x = 3x + (-10)x$

$$= (3 + (-10))x = \boxed{-7x}$$

Example 3 Simplify $7x + 4y$

Solution: Different variables so doesn't simplify.

- There are more examples on p178, 179.

6.5 Linear equations in one variable

If an equation only has linear terms with the same variable on both sides, and numbers, then it is a linear equation in one variable. These are the simplest types of equations.

For example $3x = 15$ is one and

$-2x + 6x + 2 = x - 4$ is another.

Definition: A solution of a linear equation is a number you can substitute for the variable to make the equation true.

Example ① Is 4 a solution to $3x = 15$? How about 5?

Solution: If we substitute 4 we get

$$3(4) = 15$$
$$12 = 15 \quad \text{false}$$

So 4 is not a solution. But $3(5) = 15$ is true so 5 is a solution.

Example ② Is -2 a solution to

$$-2x + 6x + 2 = x - 4 \text{ ?}$$

Solution: Substitute

$$-2(-2) + 6(-2) + 2 = (-2) - 4 \text{ ?}$$

$$4 + (-12) + 2 = (-2) + (-4) \text{ ?}$$

$$-6 = -6 \ ? \quad \text{true}$$

Answer: -2 is a solution.

- More examples of checking solutions p179, 180.

Note that in example 2 we could have simplified the two linear terms on the left

$$\underbrace{-2x + 6x}_{4x} + 2 = x - 4 \qquad \leftarrow (-2)+6 = 4$$

to get $\quad 4x + 2 = x - 4$

We say this equation is <u>equivalent</u> to the original because it will have the same solutions.

We will see that any linear equation in x is equivalent to (can be simplified to)

$$\boxed{\begin{array}{c} ax + b = c \\ \text{for numbers } a, b, c \text{ with } a \neq 0. \end{array}}$$

Example 3 Can you spot the solution

to $\quad -5x + 1 = 6$?

Try it...

Solving linear equations

There are two basic steps to solving a linear equation. First, (A) add the same number to both sides and (B) divide (or multiply) both sides by the same number.

Example (4) Use step (A) to solve $x - 5 = 6$

Solution: If $x - 5 = 6$ is true, then adding 5 to both sides will keep it true

$$\begin{array}{rcl} x - 5 & = & 6 \\ +5 & & +5 \\ \hline x & = & 11 \end{array}$$

Answer: $\boxed{x = 11 \text{ is the solution}}$ (check $(11) - 5 = 6$?)

Note that in the last example we could have added (or subtracted) any number on both sides, but only $+5$ gets to the solution.

Example (5) Use step (B) to solve $3x = 15$.

Solution: Divide both sides by 3 to get $1x$

$$\frac{3x}{3} = \frac{15}{3}$$

$$x = 5$$

Answer: $\boxed{x = 5 \text{ is the solution}}$ (check $3(5) = 15$ ✓)

Common mistakes

For example ④ a common mistake is

$$\begin{array}{rcl} x - 5 &=& 6 \\ -6 && -6 \\ \hline x - 11 &=& 0 \end{array}$$

We really want x alone on one side so this is not helpful.

For example ⑤ a common mistake is

$$\begin{array}{rcl} 3x &=& 15 \\ -3 && -3 \\ \hline x &=& 12 \end{array} \quad ?? \qquad \underline{\text{No}}$$

$3x - 3$ does not simplify

Example ⑥ Solve $4x + 3 = 31$

Solution: For step Ⓐ we want to add -3 to both sides first

$$\begin{array}{rcl} 4x + 3 &=& 31 \\ -3 && -3 \\ \hline 4x &=& 28 \end{array}$$

Then divide both sides by 4 in step Ⓑ

$$\frac{4x}{4} = \frac{28}{4}$$

$$x = 7$$

We find the solution is 7. Let's check it works:

$$\underbrace{\underbrace{4(7)}_{28} + 3}_{31} = 31 \ ?$$

True ✓.

Answer: $\boxed{x=7 \text{ is the solution}}$.

In example ⑥ if we try to divide by 4 first it doesn't work

$$\frac{4x+3}{4} = \frac{31}{4}$$

$$\vdots$$

??

Example ⑦ Solve $-9x+4=-14$

Solution:

Ⓐ
$$\begin{array}{rcr} -9x + 4 & = & -14 \\ -4 & & -4 \\ \hline -9x & = & -18 \end{array}$$

For step Ⓑ divide both sides by -9 (not 9)

$$\frac{-9x}{-9} = \frac{-18}{-9}$$

$$x = 2$$

Check: $\underbrace{-9(2)}_{-18} + 4 = -14 \ ?$ ✓ Answer $\boxed{x=2}$.

• See p181, 182 for more.