6.3 Functions

A very useful way to write expressions or formulas is to write them as a function (of one variable).

The expression $16t^2$ is a formula for the distance in feet an object falls in t seconds. To make this a function we give it a name, like f and write

$$\underbrace{f(t)}_{\text{"f of t"}} = 16t^2$$

Then $f(3)$ means substitute 3 for t to get $16(3)^2 = 16 \cdot 9 = 144$ and so

$$f(3) = 144$$

We can also see

$$f(0) = 0$$
$$f(1) = 16$$
$$f(2) = 64 \quad \text{"f of 2 equals 64"}$$

In this one, the input is 2 (number going in) and the output is 64 (number coming out).

For a function like this f it's important to remember that $f(t)$ does not mean multiplication, and f is not a number.

Example ② Let g be the function given by

$$g(x) = x^2 + 4x$$

Find (a) $g(0)$, (b) $g(5)$, (c) $g(-2)$

Solutions: Part (a) needs

$$g(0) = (0)^2 + 4(0) = 0 + 0 = 0$$

so $\boxed{g(0) = 0}$

In part (b), $g(5) = (5)^2 + 4(5)$
$= 25 + 4(5)$
$= 25 + 20 = 45$

so $\boxed{g(5) = 45}$

Part (c): $g(-2) = (-2)^2 + 4(-2)$
$= 4 + 4(-2)$
$= 4 + (-8) = -4$

and $\boxed{g(-2) = -4}$

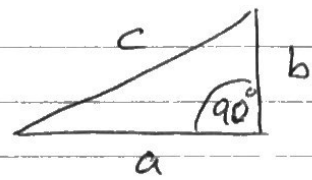
- More examples on pages 175 – 177.

Note that working out values of functions is just evaluating expressions or formulas as in the last two sections.

The course MTH 30, Precalculus is a detailed study of functions.

1.9 The Pythagorean Theorem

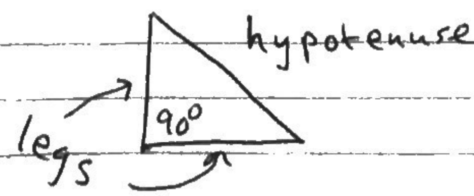
We looked at this briefly in Chapter 1 and now we look in more depth.



$$a^2 + b^2 = c^2$$

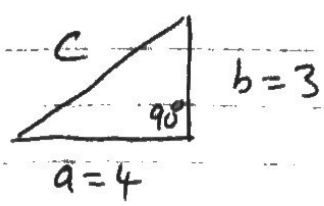
Pythagorean Theorem: In a right triangle the sum of the squares of the short sides equals the square of the long side.

A right triangle is just a triangle with a right angle (90°). The shorter sides are on each side of the right angle and are called <u>legs</u>. The longest side is opposite the right angle and called the <u>hypotenuse</u>.



<u>Example</u> ① If the legs of a right triangle are length 3 and 4, find the length of the hypotenuse.

Solution:



Use a, b for the legs

Pythagorean Theorem says $a^2 + b^2 = c^2$

So $\underbrace{(4)^2 + (3)^2}_{\underbrace{16 + 9}_{25}} = c^2 \rightarrow 25 = c^2$

means c must be 5

Answer: the hypotenuse has length 5.

To solve $25 = c^2$ we really need to take the square root of both sides

$$\sqrt{25} = \sqrt{c^2}$$
$$= c$$

and in general

$$\sqrt{a^2+b^2} = \sqrt{c^2} = c$$

so we get a nice expression (formula) for the number c

$$\boxed{c = \sqrt{a^2+b^2}}$$

hypotenuse legs

Example ② Find the hypotenuse length if the legs are of length 5 and 12.

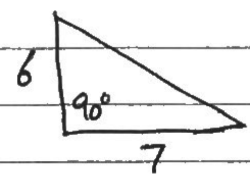
Solution: Substitute $a=5$, $b=12$ in our formula

$$c = \sqrt{a^2+b^2}$$
$$= \sqrt{5^2+12^2}$$
$$= \sqrt{25+144}$$
$$= \sqrt{169} = 13$$

Answer: Hypotenuse has length 13.

Example (3) Draw a right triangle with legs of length 6 and 7. The hypotenuse is slightly bigger than which whole number?

Solution: The triangle looks like this



Hypotenuse $c = \sqrt{a^2+b^2}$
$$= \sqrt{6^2+7^2}$$
$$= \sqrt{36+49}$$
$$= \sqrt{85}$$

This number is between $\sqrt{81} = 9$ and $\sqrt{100} = 10$

So the hypotenuse is slightly bigger than 9.

If you know one leg and the hypotenuse you can find the other leg.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ -a^2 \quad\quad & \quad -a^2 \\ \hline b^2 &= c^2 - a^2 \end{aligned}$$

subtract a^2 from both sides

$$b = \sqrt{c^2 - a^2}$$

square root of both sides

We get another formula

$$\boxed{b = \sqrt{c^2 - a^2}}$$

leg hypotenuse leg

Example ④ If the hypotenuse is 25 and one leg is 24, find the other leg.

Solution: We have $c = 25$ and $a = 24$ in the formula, looking for other leg b.

$$\begin{aligned} b &= \sqrt{(25)^2 - (24)^2} \\ &= \sqrt{625 - 576} \\ &= \sqrt{49} = 7 \end{aligned}$$

$$\begin{array}{r} \scriptstyle 2 \\ 25 \\ \times\ 25 \\ \hline 125 \\ 50\ \ \\ \hline 625 \end{array}$$

$$\begin{array}{r} \scriptstyle 1 \\ 24 \\ \times\ 24 \\ \hline 96 \\ 48\ \ \\ \hline 576 \end{array}$$

$$\begin{array}{r} 625 \\ -\ 576 \\ \hline 49 \end{array}$$

Answer:

other leg has length 7