Chapter 6 Towards algebra

In algebra we use letters like $x, y, a, b, c, \ldots$ to stand for numbers. These letters are called <u>variables</u>.

Combining variables and numbers with addition, subtraction, multiplication, division, roots and powers makes <u>algebraic expressions</u>.

For example

$$\frac{2x^2 - 3 + \sqrt{x}}{xy + 7y^3}$$

is a complicated algebraic expression.

6.1 Evaluating expressions

If you are told what numbers the variables represent then you can <u>evaluate</u> an algebraic expression.

Steps to evaluate an expression

A Replace all the variables by their numbers. Use parentheses, especially for negative numbers.

B Now use the usual order of operations to simplify the answer to a single number.

Example 1 Evaluate the expression $2x+y$ if $x=5$ and $y=3$.

Solution: Step A is called substitution

$$2x+y = 2(5)+(3)$$

means 2 is multiplying whatever x is

$$= 10+3$$

$$= \boxed{13}$$

Example 2 Evaluate x^2-3x+6 if
(a) $x=0$ (b) $x=10$ (c) $x=-4$

Solution: For part (a)

$$x^2-3x+6 = (0)^2-3(0)+6$$
$$= 0 - 3(0) + 6$$
$$= 0 - 0 + 6$$
$$= 0+6 = \boxed{6}$$

Part (b): $x^2-3x+6 = (10)^2-3(10)+6$
$$= 100 - 3(10) + 6$$
$$= 100 - 30 + 6$$
$$= 70 + 6 = \boxed{76}$$

Part (c): $x^2-3x+6 = (-4)^2-3(-4)+6$

$$= 16 - 3(-4) + 6$$

$$= 16 + 12 + 6$$
$$= \boxed{34}$$

So the expression $x^2 - 3x + 6$ equals different numbers, depending on what x is.

Variables can represent fractions and decimals.

Example ③ Evaluate $x^2 - 3x + 6$ if
(a) $x = \frac{2}{5}$ (b) $x = 1.9$

Solution: Part (a)

$$x^2 - 3x + 6 = \left(\frac{2}{5}\right)^2 - 3\left(\frac{2}{5}\right) + 6$$

$$= \frac{4}{25} - \frac{3}{1}\cdot\frac{2}{5} + \frac{6}{1}$$

LCD = 25

$$= \frac{4}{25} - \frac{6}{5} + \frac{6}{1}$$

$$= \frac{4}{25} - \frac{30}{25} + \frac{6\cdot 25}{25}$$

$$= \frac{4 - 30 + 150}{25} = \frac{124}{25}$$

$$= \boxed{4\tfrac{24}{25}}$$

Part (b)

$$x^2 - 3x + 6 = (1.9)^2 - 3(1.9) + 6$$

$(1.9)^2 = (1.9)(1.9)$

1 + 1 = 2 places

$$\begin{array}{r} \scriptstyle 8 \\ 19 \\ \times\, 19 \\ \hline 171 \\ 19 \\ \hline 361 \end{array} = 3.61$$

$3(1.9) = 5.7$

$$= 3.61 - 5.7 + 6$$

$$= 9.61 - 5.7$$

$$\begin{array}{r} \scriptstyle 8\ \ 16 \\ \cancel{9}.\cancel{6}1 \\ -\,5.70 \\ \hline 3.91 \end{array}$$

Ans $\boxed{3.91}$

It's easy to make mistakes when substituting negative numbers.

Example (4) Find $a^2 - ab - b^2$ when $a = -3$ and $b = -4$.

Solution: $a^2 - ab - b^2 = (-3)^2 - (-3)(-4) - (-4)^2$

nothing to do in the parentheses

We're using

P	( )	
E	exponents	
MD	$\times, \div$	left $\rightarrow$ right
AS	$+, -$	left $\rightarrow$ right

So exponents first $(-3)^2 = (-3)(-3) = 9$

$(-4)^2 = (-4)(-4) = 16$

get $9 - \underbrace{(-3)(-4)}_{} - 16$

M: multiply

ab meant a times b

$= \underbrace{9 - 12}_{} - 16$

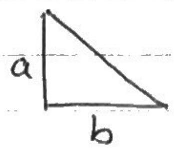
$= -3 \quad -16 = \boxed{-19}$

- Examples p169 – 171.

6.2 Using formulas

Formulas are just algebraic expressions for something we want to find.

We've already seen formulas in geometry:



area $= LW$
perimeter $= 2L + 2W$

area $= \dfrac{ab}{2}$

Example ① Find the area and perimeter of a rectangle with width 1.2, length 2.6.

Solution: Here $W = 1.2$, $L = 2.6$ and

$$\text{area} = LW = (2.6)(1.2) = 3.12$$

$$\text{perimeter} = 2L + 2W = 2(2.6) + 2(1.2)$$
$$= 5.2 + 2.4$$
$$= 7.6$$

$$\begin{array}{r} 26 \\ \times\ 12 \\ \hline 52 \\ 26 \\ \hline 312 \end{array}$$

Answer: Area is 3.12, perimeter is 7.6

Example ② The Body Mass Index (BMI) formula is

$$BMI = \frac{703w}{h^2}$$

where w = weight in pounds
and h = height in inches.
Find the BMI of someone 5 ft 10 in tall

and weighing 200 lbs.

Solution: $w = 200$ and $h = 5 \cdot 12 + 10 = 70$

Then the BMI $= \dfrac{703(200)}{(70)^2} = \dfrac{140600}{4900}$

$= \dfrac{1406}{49}$

$$
\begin{array}{r}
28.69 \\
49 \overline{)1406.00} \\
-98 \\
\hline
426 \\
-392 \\
\hline
340 \\
-294 \\
\hline
460 \\
441 \\
\hline
19
\end{array}
$$

Answer.

BMI = 28.7 rounded to nearest tenth.

A healthy BMI is < 25 so this person is overweight.

- More examples p 173 – 174.