Chapter 5 Ratio and proportion.

5.1 Ratios

As we saw in Chapter 3, a fraction $\frac{a}{b}$ means $a \div b$. Another way to think about this is that we are comparing the numbers a and b.

Definition: The ratio of a to b is written $a:b$ and means $\frac{a}{b}$.

We usually write ratios by using $\frac{a}{b}$ in lowest terms.

Example ① Suppose a team wins 15 games and loses 5. Give the ratio of wins to losses.

Solution: Wins to losses is 15 to 5 which is $\frac{15}{5}$ as a fraction. Then $\frac{15 \div 5}{5 \div 5} = \frac{3}{1}$

So the simplified answer is

Answer $\boxed{\text{The ratio of wins to losses is } 3:1}$

This is a good way to understand how well the team is doing. It wins three times as often as it loses.

Example (2) For this same team find

(a) The ratio of losses to wins.
(b) The ratio of wins to total games played.

Solution: For (a), losses to wins is 5 to 15 which is $\boxed{1:3}$

For (b), the ratio is 15 to 20 and $\frac{15 \div 5}{20 \div 5} = \frac{3}{4}$

So answer is $\boxed{3:4}$

Example (3) Write 25% as a ratio.

Solution: As a fraction $25\% = \frac{25}{100}$.

Simplify this $\frac{25 \div 25}{100 \div 25} = \frac{1}{4}$. Answer $\boxed{1:4}$

Example (4) Your giant new HDTV screen is 64 inches wide and 36 inches high. Find the ratio of width to height (called aspect ratio).

Solution: $\frac{64}{36} = \frac{64 \div 4}{36 \div 4} = \frac{16}{9}$ simplified

So the ratio is $\boxed{16:9}$

- More examples in book, p150, 151.

5.2 Proportions

We just saw in example 2 that

$$\frac{15}{20} = \frac{3}{4}$$

and it means the ratios 15:20 and 3:4 are really the same.

Definition: A proportion states that two ratios are equal and looks like $\longrightarrow$ $\frac{a}{b} = \frac{c}{d}$

So two equal (equivalent) fractions make a true proportion.

Example 5 Are these proportions true?

(a) $\frac{6}{14} = \frac{9}{21}$ (b) $\frac{5}{10} = \frac{3}{5}$

Solution: If we reduce the fractions in part (a) we see $\frac{6}{14} = \frac{6 \div 2}{14 \div 2} = \frac{3}{7}$, $\frac{9}{21} = \frac{9 \div 3}{21 \div 3} = \frac{3}{7}$

so the proportion in (a) is true.

In part (b) we can use the LCD to see the proportion is false

LCD = 10 $\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$ and $\frac{5}{10} \neq \frac{6}{10}$.

Cross products

There is an easier way to check if a proportion is true: compute the cross products and see if they are equal.

(3a) $\frac{6}{14} = \frac{9}{21}$?

$$\frac{6}{14} = \frac{9}{21}$$

Cross products
$6 \cdot 21 = 126$
$14 \cdot 9 = 126$ ✓

(3b) $\frac{5}{10} = \frac{3}{5}$?

$$\frac{5}{10} = \frac{3}{5}$$

Cross products
$5 \cdot 5 = 25$
$10 \cdot 3 = 30$ ✗

<u>Example</u> (6) Use the cross product property to see if $\frac{9}{20} = \frac{4}{9}$

Solution: The cross products are $9 \cdot 9 = 81$ and $20 \cdot 4 = 80$. Not equal so the equation is false.

- See p. 152, 153 for why this works and more examples.

Finding cross products is <u>only</u> used for checking if proportions are true.

Multiply fractions straight across $\frac{3}{4} \cdot \frac{5}{7} = \frac{15}{28}$

Solving proportions

We saw the true proportion $\frac{15}{20} = \frac{3}{4}$ earlier. If one of the numbers was missing

$$\frac{15}{20} = \frac{?}{4}$$

could we have found it? This is called solving the proportion. We usually use x to represent an unknown number.

Example ⑦ Solve $\frac{15}{20} = \frac{x}{4}$

Solution: Set the cross products equal

$$15 \cdot 4 = 20x \quad \text{so} \quad \underbrace{60 = 20x}_{\text{for this to be true } x \text{ must be } 3}$$

Answer $\boxed{x = 3}$

(We say 3 is the solution.)

Second solution: It's easier if we simplify the left side $\frac{15}{20}$ first – $\frac{15 \div 5}{20 \div 5} = \frac{3}{4}$.

The proportion

$$\frac{15}{20} = \frac{x}{4} \quad \text{becomes} \quad \frac{3}{4} = \frac{x}{4}$$

and now its clear that x must be 3.

Example 8 Solve the proportion $\frac{6}{12} = \frac{10}{X}$

Solution: First $\frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2}$

and we get $\frac{1}{2} = \frac{10}{X}$. Cross products: $\underset{X}{\underset{\shortparallel}{1 \cdot X}}$, $\underset{20}{\underset{\shortparallel}{2 \cdot 10}}$

We want the cross products to be equal

$$X = 20$$

so the solution is $\boxed{X = 20}$.

Example 9 Solve $\frac{33}{B} = \frac{3}{5}$

Solution: The cross products are

$33 \cdot 5 = 165$ and $B \cdot 3 = 3B$

For the cross products to be equal

want $165 = 3B$

to find B we can divide both sides by 3

$$\underbrace{\frac{165}{3}} = \underbrace{\frac{3B}{3}}$$
$$55 = B$$

Solution $\boxed{B = 55}$

- More examples p153–155.