4.8 Division of a decimal by a whole number

We already looked at division of whole numbers in section 1.5. For example:

① what is $13 \div 5$? Solution

$$5 \overline{\smash{)}13} \quad \leftarrow \text{quotient } 2$$
$$\underline{-10}$$
$$3 \quad \leftarrow \text{remainder}$$

So one answer is $\boxed{2 \text{ R } 3}$

Another answer is $\boxed{2\frac{3}{5}}$ as a mixed number.

We see a third answer next as a decimal.

13 can be written as 13.0 or 13.00000 by adding insignificant zeros and this lets us continue the division

put decimal point here

$$2.\ \ \ $$
$$5 \overline{\smash{)}13.0}$$
$$\underline{-10}$$
$$3$$

$$\longrightarrow$$

$$2.6$$
$$5 \overline{\smash{)}13.0}$$
$$\underline{-10} \downarrow$$
$$3\,0$$
$$\underline{-3\,0}$$
$$0 \quad \leftarrow \text{stops when you get zero}$$

So as a decimal
$$13 \div 5 = \boxed{2.6}$$

Note that $2.6 = 2\frac{6}{10}$ and $\frac{6 \div 2}{10 \div 2} = \frac{3}{5}$

so $2.6 = 2\frac{3}{5}$ and this agrees with our second answer.

Usually the question will specify what kind
of answer is wanted:

Find $13 \div 5$ and give the answer as

(a) a quotient and remainder $\boxed{2 \text{ R } 3}$
(b) a mixed number $\boxed{2\,{}^3\!/_5}$
(c) a decimal $\boxed{2.6}$

Example ② Find $3 \div 8$ as a decimal.

Solution:

$$\begin{array}{r} 0 \\ 8\overline{)3} \\ 0 \\ \hline 3 \end{array}$$

add decimal points,
insignificant zeros
and keep going

$$\begin{array}{r} 0.3 \\ 8\overline{)3.00} \\ \underline{0}\downarrow \\ 30 \\ \underline{-24} \\ 6 \end{array} \qquad \longrightarrow$$

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ \underline{0}\downarrow \\ 30 \\ \underline{-24}\downarrow \\ 60 \\ \underline{-56}\downarrow \\ 40 \\ \underline{-40} \\ 0 \end{array} \quad \text{stop}$$

Answer $3 \div 8 = \boxed{0.375}$

$\left(\text{also } 3 \div 8 = 0 \text{ R } 3 \quad \text{and} \quad \dfrac{3}{8}\right).$

Dividing a decimal by a whole number works the same way — add insignificant zeros and line up the decimal points.

Example ③   Find $7.8 \div 4$

Since the question contains a decimal we will find the answer as a decimal.

Solution:

$$4 \overline{\smash{)}7.80}$$ ← set up — can add more zeros if needed

$$
\begin{array}{r}
1. \\
4\,\overline{\smash{)}\,7.80} \\
-4 \downarrow \\
\hline
3\;8
\end{array}
\longrightarrow
\begin{array}{r}
1.9 \\
4\,\overline{\smash{)}\,7.80} \\
-4 \downarrow \\
\hline
3\;8 \\
-3\,6 \downarrow \\
\hline
2\,0
\end{array}
\longrightarrow
\begin{array}{r}
1.95 \\
4\,\overline{\smash{)}\,7.80} \\
-4 \downarrow \\
\hline
3\;8 \\
-3\,6 \downarrow \\
\hline
2\;0 \\
-2\,0 \\
\hline
0
\end{array}
$$

Answer: $7.8 \div 4 = \boxed{1.95}$

Of course your calculator or phone will give this answer, but we want to do it by hand.

We can also see the answer makes sense —

If it was $8 \div 4$ then the answer is 2.

7.8 is close to 8   so $7.8 \div 4$ should be close to 2 which it is.

- See examples on pages 134-136.

The decimal division stops when we get a zero in the remainder place. But what happens if we never get a zero?

There are two options:

(A) Find the repeating pattern and use the bar notation - for example

$4.62\overline{78}$ means $4.62\ 78\ 78\ 78\ 78...$

(B) Stop at a certain point and round your answer. For example divide out to 3 places and round your answer to the nearest hundredth (2 places).

Example ④ Find $5.9 \div 3$ and write your answer
(a) as a repeating decimal
(b) rounded to the nearest thousandth.

Solutions: Set up the division

$$3\overline{)5.9000}$$

and get

$$
\begin{array}{r}
1.96 \\
3\overline{)5.9000} \\
-3\downarrow \\
\hline
2\,9 \\
-2\,7\downarrow \\
\hline
2\,0 \\
-1\,8 \\
\hline
2
\end{array}
$$

after this we see the same thing happening each time

So the answer is 1.966666666 --- with 6s going forever.

Write $5.9 \div 3 =$ $\boxed{1.9\overline{6}}$ answer to (a)

For part (b) remember rounding from section 4.4

$$1.96\boxed{6}66\cdots$$

↗ rounding place thousandths

↖ next place in range 5-9 so increase digit in rounding place

So $5.9 \div 3 = \boxed{1.967}$ rounded to nearest thousandth.

Note that $1.9\overline{6}$ is a better answer because it is <u>exact</u>. 1.967 is an approximation.

• More examples p136-138.

4.9 Division of a decimal by a decimal

In the last section we divided a decimal by a whole number. How do you divide a decimal by a decimal?

For a simple example, let's look at $1.2 \div 0.3$

How many times does 0.3 fit into 1.2? You might see the answer is 4 times.

Can also

write $1.2 \div 0.3 = \dfrac{1.2}{0.3} = \dfrac{1.2 \times 10}{0.3 \times 10} = \dfrac{12}{3}$

$$= \boxed{4}.$$

Example ⑤ Find $0.342 \div 0.09$

Solution: We use the same trick to get an equivalent question where we are dividing by a whole number:

$$\frac{0.342}{0.09} = \frac{0.342 \times 100}{0.09 \times 100} = \frac{34.2}{9}$$

then

$$9\overline{)34.2} \qquad \text{as before}$$

$$
\begin{array}{r}
3.8 \\
9\overline{)34.2} \\
-27\downarrow \\
\hline
72 \\
-72 \\
\hline
0
\end{array}
$$

Answer

$0.342 \div 0.09 = \boxed{3.8}$.

We multiplied top and bottom by 100 because that was the smallest power of ten that makes the bottom a whole number.

Remember that multiplying by powers of 10 moves the decimel point right

$0.09 \times 100 = 9. = 9 \qquad 0.342 \times 100 = 34.2$

Rule to divide two decimals

- First multiply both numbers by a power of 10 that makes the number you are dividing by a whole number (so move the decimal points the same distance).

- Now divide by the whole number, lining up the decimal point of the answer as usual.

Example ⑥ Compute $6 \div 0.7$ and round your answer to the nearest thousandth.

Solution: 0.7 is not a whole number so multiply by 10 and do same for the other number $\dfrac{6}{0.7} = \dfrac{6 \times 10}{0.7 \times 10} = \dfrac{60}{7}$

$0.7 \times 10 = 7$ $\quad 6 = 6.0 \times 10 = 60$

Now $7\overline{)60.0} \longrightarrow$

We have enough places to round to nearest thousandth

$8.57\boxed{1}4 \longrightarrow \boxed{8.571}$

keep $\qquad$ Answer.

$$
\begin{array}{r}
8.5714 \\
7\overline{)60.0000} \\
-56 \downarrow \\
\hline
40 \\
-35 \downarrow \\
\hline
50 \\
-49 \downarrow \\
\hline
10 \\
-7 \\
\hline
30
\end{array}
$$

Example ⑦ Find $-0.000319 \div 0.011$

Solution: Multiply both by 10^3 in the first step

$$= -0.000319 \div 0.011$$

and also note that the answer will be negative

$$= -(0.319 \div 11)$$

We get

$$11 \overline{)0.\overset{\bullet}{3}19}$$

and

$$\begin{array}{r} 0.029 \\ 11 \overline{)0.319} \\ -22\downarrow \\ \hline 99 \\ -99 \\ \hline 0 \end{array}$$

so answer is

$$\boxed{-0.029}$$

You could also check this answer by multiplying 0.011 back on

$$(-0.029)(0.011)$$

should equal the first number

$$-0.000319$$

(it does)