

4.10 Scientific notation

Scientific notation is a convenient way to express very big numbers and very small decimals.

An example of scientific notation is 7.2×10^6

Written out this is

$$\underline{7,200,000} \times 10^6 = 7200000 \quad \text{big number}$$

Another example is 4.52×10^{-3}

(remember that $4.52 \times 10^{-3} = 4.52 \div 10^3$)

Written out this is

$$\underline{0.00452} \times 10^{-3} = 0.00452 \quad \begin{array}{l} \text{very} \\ \text{small} \end{array}$$

For scientific notation	$\underline{7.2} \times 10^6$	$\underline{4.52} \times 10^{-3}$
	between 1 and 10	between 1 and 10

The first number must be 1 or larger and < 10 . Multiply by a power of 10. This power can be positive or negative (or zero) and is called the order of magnitude.

Example ① Convert 302.9 into scientific notation.

Solution: To get a number between 1 and 10 we must move the decimal point

302.9 2 places

3.029 is 100 times smaller

so $302.9 = \boxed{3.029 \times 10^2}$

Example ② Convert 0.000000015 to scientific notation.

Solution: Move decimal point

0.000000015 7 places

1.5 is 10^7 times bigger

so $0.000000015 = 1.5 \div 10^7$
 $= \boxed{1.5 \times 10^{-7}}$

Formula for scientific notation

$$\boxed{a \times 10^N}$$

with $1 \leq a < 10$

N = order of magnitude

For big numbers N is positive.

For small numbers N is negative.

4.11 Percents, conversions

2.

If you see an ad "SALE 30% OFF!"

that 30% (thirty percent) is really the

fraction $\frac{30}{100}$. And that's what percent

means in Latin - put over 100.

Example ③ Convert these percents to fractions and simplify

(a) 50% (b) 100% (c) 6% (d) 260%

Solutions:

$$(a) \ 50\% = \frac{50}{100} = \frac{50 \div 50}{100 \div 50} = \boxed{\frac{1}{2}}$$

$$(b) \ 100\% = \frac{100}{100} = \boxed{1}$$

$$(c) \ 6\% = \frac{6}{100} = \frac{6 \div 2}{100 \div 2} = \boxed{\frac{3}{50}}$$

$$(d) \ 260\% = \frac{260}{100} = \frac{260 \div 10}{100 \div 10} = \frac{26 \div 2}{10 \div 2} = \boxed{\frac{13}{5}}$$

Note that dividing by $100 = 10^2$ moves the decimal point 2 places left. So converting the percents in example ③ to decimals is very easy

$$\underline{50\%} = 0.5$$

$$\underline{100\%} = 1$$

$$\underline{06\%} = 0.06$$

$$\underline{260\%} = 2.6$$

We see there are lots of ways to write one half (or any fraction)

$$\begin{array}{ccccccc} \frac{1}{2} & = & 50\% & = & \frac{50}{100} & = & 0.5 \\ \uparrow & & \uparrow & & & & \uparrow \\ \text{fraction} & & \text{percent} & & & & \text{decimal} \end{array}$$

It is useful to be able to switch between these three representations

Percent \rightarrow fraction $13\% = \frac{13}{100}$

Percent \rightarrow decimal $\underline{13}\% = 0.13$

Decimal \rightarrow Percent $0.\underline{13} = 13\%$

Decimal \rightarrow Fraction $0.13 = \frac{13}{100}$

Starting with a fraction we must convert to a decimal first with a division.

Example (4) Convert $\frac{1}{5}$ to a decimal and then a percent.

Solution: $\frac{1}{5} = 1 \div 5$ so $5 \overline{)1.0}$

and
$$\begin{array}{r} 0.2 \\ 5 \overline{)1.0} \\ \underline{-1.0} \\ 0 \end{array} \quad \text{then } \frac{1}{5} = \boxed{0.2} = 0.20 = \boxed{20\%}$$

fraction $\xrightarrow{\text{divide}}$ decimal $\xrightarrow{\text{}} \text{percent}$

Example (5) Convert $\frac{3}{7}$ to a percent, rounded to the nearest tenth of a percent.

Solution: First convert to a decimal

$$7 \overline{) 3.0000}$$

$$\begin{array}{r} 0.4285 \\ 7 \overline{) 3.0000} \\ \underline{-28} \downarrow \\ 20 \downarrow \\ \underline{-14} \downarrow \\ 60 \downarrow \\ \underline{-56} \\ 40 \\ \underline{-35} \\ 5 \end{array}$$

$$\text{so } \frac{3}{7} = 0.4285 \dots$$

$$= 42.85\%$$

↑
tenth of a
percent place

$$42.\boxed{8}5$$

↑
rounding
place

↑ next is in
range 5-9
so increase

$$\text{Answer: } \frac{3}{7} = \boxed{42.9\%}$$

rounded to nearest
tenth of a %.

In this example we needed to do the decimal division to 4 places to have enough places for the rounding.

4.12 Fractional parts of numbers

Suppose a coat sells for \$40. How much is it reduced by in a 30% off sale?

In other words, what is 30% of 40?

To compute this we convert 30% into a number (a fraction or a decimal) and remember that "of" means multiply.

$$\text{So } 30\% \text{ of } 40 = \frac{30}{100} \cdot \frac{40}{1} = \frac{1200}{100} = 12$$

$$\text{or } = (0.3)(40) = 12$$

Either way the sale price of the coat is \$28.

Example (6) Find 6% of 90

$$\text{Solution: Want } \frac{6}{100} \cdot \frac{90}{1} = \frac{540}{100} = \boxed{5.4}$$

• See p 147.

Example (7) Compute five thirds of 120.

$$\text{Solution: } \frac{5}{3} \cdot \frac{120}{1} = \frac{5}{3} \cdot \frac{3 \cdot 40}{1} = \boxed{200}$$

Example (8) What is 9.2% of 4.7?

$$\text{Solution: } \underbrace{09.2\%}_{\text{check}} = 0.092$$

$$\text{So we need } (0.092)(4.7) = \boxed{0.4324}$$