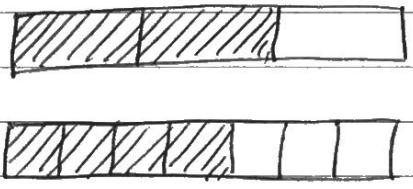
3.9 Comparing fractions

If you have two fractions, how do you know which is bigger and which is smaller?

Example ① Draw the rectangles for $\frac{2}{3}$, $\frac{4}{7}$ and see which is smaller.

Solution:



It looks like $\frac{4}{7}$ is smaller than $\frac{2}{3}$.

A better way to compare is to use a common denominator. The LCD of $\frac{2}{3}$ and $\frac{4}{7}$ is 21

$$\frac{2}{3} = \frac{2 \cdot 7}{3 \cdot 7} = \frac{14}{21}$$

$$\frac{4}{7} = \frac{4 \cdot 3}{7 \cdot 3} = \frac{12}{21}$$

This shows that $\frac{4}{7} < \frac{2}{3}$ is definitely true.

Method: To compare two or more fractions in size use the LCD.

Example ② Put $\frac{3}{4}, \frac{2}{5}, \frac{1}{2}$ in increasing order.

Increasing order means they should be increasing in size so want small, middle, biggest.

Solution: the LCM of 4, 5, 2 is 20

Multiples of 4: 4, 8, 12, 16, 20, 24, - -
" of 5: 5, 10, 15, 20, 25, - -
" of 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, - -

Write $\frac{3}{4} = \frac{3 \cdot 5}{4 \cdot 5} = \frac{15}{20}$ biggest

$\frac{2}{5} = \frac{2 \cdot 4}{5 \cdot 4} = \frac{8}{20}$ smallest

$\frac{1}{2} = \frac{1 \cdot 10}{2 \cdot 10} = \frac{10}{20}$ middle

So write the answer with the original fractions, and use $<$ to indicate increasing

Answer $\boxed{\frac{2}{5} < \frac{1}{2} < \frac{3}{4}}$

- More examples p 98, 99.

3.10 Division of Fractions

The last operation we need for fractions is division. This can be confusing.

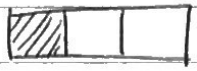
First we look at reciprocals.

To get the reciprocal of a number you just put 1 over it, so the

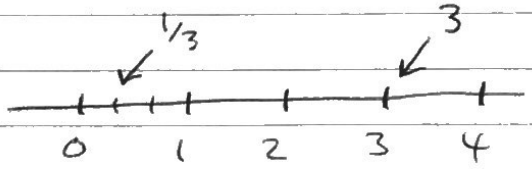
reciprocal of 3 is $\frac{1}{3}$

These are very different numbers

$3 =$ 

$\frac{1}{3} =$ 

Or, on the number line



If you multiply a number by its reciprocal

$$3 \cdot \frac{1}{3} = \frac{3}{1} \cdot \frac{1}{3} = \frac{3 \cdot 1}{1 \cdot 3} = \frac{3}{3} = 1$$

you always get 1.

Definition: two numbers are reciprocal if their product is 1.

Example (3) Find reciprocals of (a) 2, (b) $\frac{1}{3}$, (c) $\frac{4}{5}$.

Solutions: (a) $\frac{1}{2}$ because $\frac{1}{2} \cdot \frac{2}{1} = 1$

(b) 3 because $\frac{3}{1} \cdot \frac{1}{3} = 1$

(c) $\frac{5}{4}$ because $\frac{5}{4} \cdot \frac{4}{5} = \frac{20}{20} = 1$.

We see that the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$

$a \neq 0, \quad b \neq 0.$

Example (4) Compute $6 \div 2$ and $6 \cdot \frac{1}{2}$.

Solution: $6 \div 2 = 3$ and $\frac{6}{1} \cdot \frac{1}{2} = \frac{6}{2} = 3$ also.

This last example leads to the rule:

> Dividing by a number is always the same as multiplying by its reciprocal

Example (5) Find $3 \div \frac{1}{4}$

Solution: the reciprocal of $\frac{1}{4}$ is 4, so by the rule $3 \div \frac{1}{4} = 3 \cdot 4 = \boxed{12}$.

Another way to see it —

$$3 \div \frac{1}{4} = \frac{3}{\frac{1}{4}} = \frac{3 \cdot 4}{\frac{1}{4} \cdot 4} = \frac{12}{1} = \boxed{12}.$$

So the rule for dividing fractions is

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

keep change flip

Example (6) Compute $\frac{2}{3} \div \frac{3}{5}$

Solution: $\frac{2}{3} \div \frac{3}{5} = \frac{2}{3} \cdot \frac{5}{3} = \boxed{\frac{10}{9}}$.

Note that we cannot cancel 3s here. Precancellation is only for multiplying fractions.

Example (7) Calculate: $15 \div \frac{3}{2}$

Solution: $\frac{15}{1} \div \frac{3}{2} = \frac{15}{1} \cdot \frac{2}{3} = \frac{30}{3} = \boxed{10}$.

can also precancel here

$= \frac{\cancel{3} \cdot 5}{1} \cdot \frac{2}{\cancel{3}} = \boxed{10}$.

Example (8) Find $\frac{8}{15} \div \frac{16}{21}$.

Solution: Get $\frac{8}{15} \cdot \frac{21}{16}$ first (keep, change, flip).

Easiest next to precancel

$$\frac{8}{15} \cdot \frac{21}{16} = \frac{8}{3 \cdot 5} \cdot \frac{3 \cdot 7}{2 \cdot 8} = \frac{7}{5 \cdot 2} = \boxed{\frac{7}{10}}.$$

cancel 8s, 3s

Multiplying and dividing mixed numbers

For mixed numbers you must convert them first into improper fractions and then multiply fractions $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

or divide fractions $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$

as usual.

- See examples p 102.

Comparing fraction operations

Find (A) $\frac{3}{5}+\frac{1}{2}$ (B) $\frac{3}{5}-\frac{1}{2}$ (C) $\frac{3}{5}\cdot\frac{1}{2}$ (D) $\frac{3}{5}\div\frac{1}{2}$

add subtract multiply divide

Solutions

(A) Need LCD = 10 $\quad \frac{3}{5}+\frac{1}{2}=\frac{3\cdot2}{5\cdot2}+\frac{1\cdot5}{2\cdot5}$

$=\frac{6}{10}+\frac{5}{10}=\boxed{\frac{11}{10}}$

(B) Need LCD = 10 $\quad \frac{3}{5}-\frac{1}{2}=\frac{6}{10}-\frac{5}{10}=\boxed{\frac{1}{10}}$

(C) Don't need LCD $\quad \frac{3}{5}\cdot\frac{1}{2}=\frac{3\cdot1}{5\cdot2}=\boxed{\frac{3}{10}}$

(D) Don't need LCD $\quad \frac{3}{5}\div\frac{1}{2}=\frac{3}{5}\cdot\frac{2}{1}=\boxed{\frac{6}{5}}$.

We only need the LCD when adding, subtracting (or comparing) fractions.

In parts (A) and (D) the answers were improper fractions. You can convert these into mixed numbers, but this is not necessary:

$\frac{11}{10}=1\frac{1}{10}$, $\quad \frac{6}{5}=1\frac{1}{5}$.

3.11 Mixed numbers and mixed units.

When multiplying mixed numbers (or dividing) you must convert them to improper fractions first, for example

$$\left(2\tfrac{1}{3}\right)\left(3\tfrac{1}{4}\right) = \frac{7}{3}\cdot\frac{13}{4} = \boxed{\frac{91}{12}}$$

$$12\overline{)91}\quad 7,\ 84,\ 7 \qquad \text{or} \qquad = \boxed{7\tfrac{7}{12}}$$

(Not $6\frac{1}{12}$!)

For adding or subtracting mixed numbers you can convert to improper fractions or leave them mixed.

Example (9) Add $2\frac{1}{3} + 3\frac{1}{4}$

First solution: Equals $\frac{7}{3} + \frac{13}{4}$

LCD = 12 $= \frac{7\cdot4}{3\cdot4} + \frac{13\cdot3}{4\cdot3} = \frac{28}{12} + \frac{39}{12}$

$$\begin{array}{r} {\scriptstyle 1\ \ \ \ \ \ } \\ 28 \\ +39 \\ \hline 67 \end{array} \qquad 12\overline{)67}\ \ 5,\ -60,\ 7 \qquad = \boxed{\frac{67}{12}} \qquad = \boxed{5\tfrac{7}{12}}.$$

Second solution: Mixed numbers

really mean $2 + \frac{1}{3} + 3 + \frac{1}{4} = 5 + \frac{1}{3} + \frac{1}{4}$

$= 5 + \frac{4}{12} + \frac{3}{12} = 5 + \frac{7}{12} = \boxed{5\tfrac{7}{12}}$.

- See pages 103–105 for more.

Adding and subtracting in mixed units is similar.

Example (10) Find 3ft 5in + 6ft 8in.

Remember there are 12 inches in 1 foot.

Solution: Vertically

$$\begin{array}{r} 3\text{ft } 5\text{in} \\ +\ 6\text{ft } 8\text{in} \\ \hline 9\text{ft } 13\text{in} \end{array}$$

$$= \boxed{10\text{ft } 1\text{in}}$$

carry 1 foot

Example (11) Subtract 4ft 7in from 8ft 3in.

Solution: Vertically

$$\begin{array}{r} 8\text{ft } 3\text{in} \\ -\ 4\text{ft } 7\text{in} \\ \hline \end{array}$$

↑ 7 is too big so borrow

$$= \begin{array}{r} {\scriptstyle 7} \quad {\scriptstyle 15} \\ \cancel{8}\text{ft } \cancel{3}\text{in} \\ -\ 4\text{ft } 7\text{in} \\ \hline 3\text{ft } 8\text{in} \end{array}$$

(Check that 3ft 8in + 4ft 7in = 8ft 3in again)

So answer is $\boxed{3\text{ft } 8\text{in}}$.

- More examples on p106, 107, 108 in book.