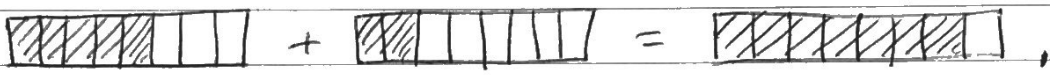
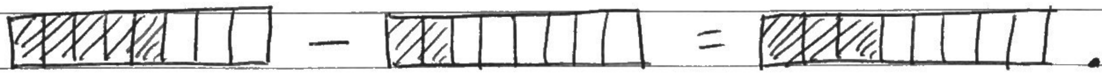
3.8 Adding and subtracting fractions

Adding and subtracting fractions is easy when the denominators are the same:

$$\frac{5}{8} + \frac{2}{8} = \frac{7}{8}$$



$$\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$$



The rules are simple: $\boxed{\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}}$ $\boxed{\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}}$

Example 1 Find $\frac{3}{10} + \frac{1}{10}$.

Solution: $\frac{3}{10} + \frac{1}{10} = \frac{3+1}{10} = \frac{4}{10} = \frac{4 \div 2}{10 \div 2} = \boxed{\frac{2}{5}}$

We always want the answer in lowest terms.

- More examples p92.

Avoid the common mistake: $\frac{3}{10} + \frac{1}{10} = \frac{4}{20}$ No

Fractions with the same denominators are called like fractions.

Fractions with different denominators are called unlike fractions.

Adding and subtracting unlike fractions is harder.

Example ② find $\frac{5}{8} + \frac{1}{4}$.

Solution: There is no way to do this directly. To use the rule $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

we need a common denominator.

Remember our other rule $\boxed{\frac{a}{b} = \frac{a \cdot c}{b \cdot c}}$

which lets us write fractions in different ways. Here $\frac{1}{4} = \frac{1 \cdot 2}{4 \cdot 2} = \frac{2}{8}$ so that

$$\frac{5}{8} + \frac{1}{4} = \frac{5}{8} + \frac{2}{8} = \boxed{\frac{7}{8}}.$$

(Same as our very first example.)

Very common mistake

$$\frac{5}{8} + \frac{1}{4} = \frac{5+1}{8+4} = \frac{6}{12} = \frac{1}{2} \quad \text{No!}$$

Rule: To add or subtract unlike fractions we need a common denominator.

Example ③ Add: $\frac{1}{2} + \frac{1}{3}$

Solution – use $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ to get equivalent fractions with the same denominator.

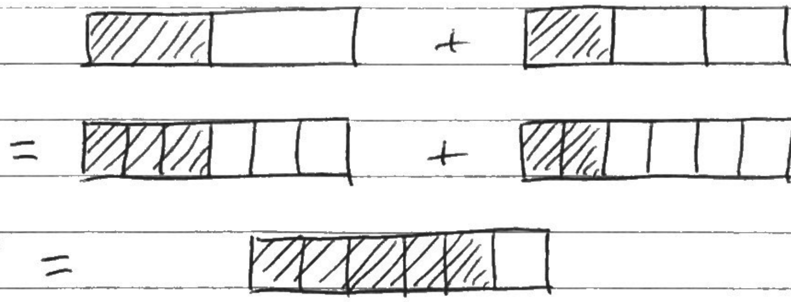
$$\frac{1}{2} + \frac{1}{3} = \frac{1\cdot 3}{2\cdot 3} + \frac{1\cdot 2}{3\cdot 2}$$

$$= \frac{3}{6} + \frac{2}{6} = \frac{3+2}{6} = \boxed{\frac{5}{6}}$$

Answer

Common denominator is 6.

Rectangle version:



Example (4) Compute $\frac{3}{7} - \frac{2}{5}$

Solution: We can get a common denominator of 35. Use $\frac{3}{7} = \frac{3\cdot 5}{7\cdot 5} = \frac{15}{35}$

and $\frac{2}{5} = \frac{2\cdot 7}{5\cdot 7} = \frac{14}{35}$

So $\frac{3}{7} - \frac{2}{5} = \frac{15}{35} - \frac{14}{35} = \frac{15-14}{35} = \boxed{\frac{1}{35}}$.

Example (5) Find $\frac{3}{10} + \frac{1}{4}$

First solution: Use the common denominator 40

$$\frac{3}{10} + \frac{1}{4} = \frac{3\cdot 4}{10\cdot 4} + \frac{1\cdot 10}{4\cdot 10} = \frac{12+10}{40}$$

cancel twos $\longrightarrow = \frac{22}{40} = \boxed{\frac{11}{20}}$.

Second solution: There is a smaller common denominator we can use 20

$$\frac{3}{10} + \frac{1}{4} = \frac{3 \cdot 2}{10 \cdot 2} + \frac{1 \cdot 5}{4 \cdot 5} = \frac{6}{20} + \frac{5}{20} = \boxed{\frac{11}{20}}$$

Same answer, but this was better because used smaller numbers.

Is there an even smaller common denominator we can use to add $\frac{3}{10} + \frac{1}{4}$?

If you look at the multiples of the denominators

multiples of 10: 10, 20, 30, 40, 50, 60, ...

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, ...

we see that 20 is the smallest possibility.

So 20 is the least common multiple (LCM) of 10 and 4 means that

20 is the least common denominator (LCD)

of the fractions $\frac{3}{10}$ and $\frac{1}{4}$.

Example ⑥ Find $\frac{1}{10} + \frac{3}{4} - \frac{7}{20}$

Solution: The LCD is 20 so

$$\frac{1}{10} + \frac{3}{4} - \frac{7}{20} = \frac{1 \cdot 2}{10 \cdot 2} + \frac{3 \cdot 5}{4 \cdot 5} - \frac{7}{20}$$

$$= \frac{2}{20} + \frac{15}{20} - \frac{7}{20}$$

$$= \frac{2 + 15 - 7}{20}$$

$$= \frac{17-7}{20} = \frac{10}{20} = \frac{10 \div 10}{20 \div 10} = \boxed{\frac{1}{2}}.$$

Least common multiples (LCMs)

Remember the multiples of a number m are $1 \cdot M$, $2 \cdot M$, $3 \cdot M$, $4 \cdot M$, ----

eg.

multiples of 6: 6, 12, 18, 24, 30, ---

$$24 \xrightarrow{+6} 30$$

You can also get them by adding 6 each time.

multiples of 8: 8, 16, 24, 32, 40, ---

Can see the LCM of 6 and 8 is 24.

(Don't confuse with the GCF - GCF of 6 and 8 is 2.)

Example (7) Find the LCM of 6, 9 and 12.

Solution:

multiples of 6: 6, 12, 18, 24, 30, 36, 42, ---

multiples of 9: 9, 18, 27, 36, 45, ---

multiples of 12: 12, 24, 36, 48, 60, ---

The smallest number in all three lists is $\boxed{36}$ so that's the LCM.

Second solution: use the prime factorizations of the numbers

$$6 = 2 \cdot 3$$
$$9 = 3 \cdot 3$$
$$12 = 2 \cdot 2 \cdot 3$$

To be a multiple of all three numbers you need at least $2 \cdot 2$ and $3 \cdot 3$

so $LCM = 2 \cdot 2 \cdot 3 \cdot 3 = \boxed{36}$.

<u>Example</u> (8) Find the LCM of 21 and 35.

Solution: $21 = 3 \cdot 7$ and $35 = 5 \cdot 7$

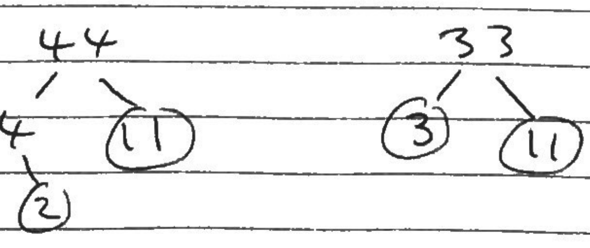
Any common multiple must have at least 3, 5, 7 as factors. $LCM = 3 \cdot 5 \cdot 7 = \boxed{105}$.

- See pages 94, 95 in book.

<u>Example</u> (9) Find the LCD for $\frac{1}{44}$ and $\frac{32}{33}$.

Use this LCD to find $\frac{1}{44} + \frac{32}{33}$.

Solution:



So $44 = 2 \cdot 2 \cdot 11$ and $33 = 3 \cdot 11$

The LCM needs $2\cdot 2$, 3, 11

So its $2\cdot 2\cdot 3\cdot 11 = 12\cdot 11 = 132$.

The LCD is $\boxed{132}$.

Now, to do the addition using this least common denominator we need

$$\frac{1}{44} = \frac{1\cdot \ldots}{44\cdot \ldots} = \frac{?}{132}$$

$$\frac{32}{33} = \frac{32\cdot \ldots}{33\cdot \ldots} = \frac{?}{132}$$

What do you multiply 44 by to get 132?
Or how many times does 44 fit into 132?
Or what is $132 \div 44$. Answer is 3.

Easy way to see $44 = 2\cdot 2\cdot 11$, $132 = 2\cdot 2\cdot \underline{\underline{3}}\cdot 11$.

So $\frac{1}{44} = \frac{1\cdot 3}{44\cdot 3} = \frac{3}{132}$

also $\frac{32}{33} = \frac{32\cdot 4}{33\cdot 4} = \frac{128}{132}$ $(33 = 3\cdot 11)$

and $\frac{1}{44} + \frac{32}{33} = \frac{3}{132} + \frac{128}{132} = \boxed{\frac{131}{132}}$.

- More examples p 97.