3.6 Prime factorization + GCF.

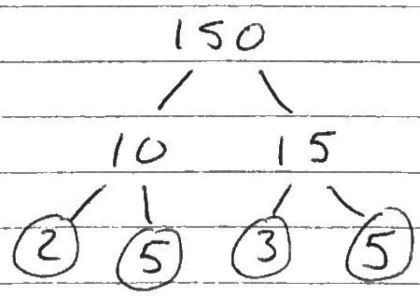
A number like 10 factors into a product of two smaller numbers $10 = 2 \cdot 5$ so 10 is called composite. A number like 11 which doesn't factor into smaller numbers is prime.

2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, ...

primes circled, others composite
(the number 1 is special – neither prime nor composite).

Example ① Factor 150 into the smallest numbers possible.

Solution: We can start with $150 = 10 \cdot 15$ and keep going $10 = 2 \cdot 5$, $15 = 3 \cdot 5$.
Draw a factor tree:



$150 = 2 \cdot 3 \cdot 5 \cdot 5$

$= \boxed{2 \cdot 3 \cdot 5^2}$

Circle the prime numbers at the bottom. We have found the prime factorization of 150.

Example ② Is 31 a prime number?

Solution: Let's see if it has any prime

factors: divide 31 by 2

$$\begin{array}{r} 15 \\ 2\overline{)31} \\ -2 \\ \hline 11 \\ -10 \\ \hline 1 \end{array}$$

$= 15\ R\ 1$

$\leftarrow$ not zero so 2 not a factor of 31

divide 31 by 3

$$\begin{array}{r} 10 \\ 3\overline{)31} \\ -3 \\ \hline 01 \\ -0 \\ \hline 1 \end{array}$$

$= 10 R 1$

3 not a factor

divide 31 by 5

$$\begin{array}{r} 6 \\ 5\overline{)31} \\ -30 \\ \hline 1 \end{array}$$

$= 6\ R\ 1$

5 not a factor

Since $7^2 = 49 > 31$ no need to check higher primes. If 7 was a factor of 31 then it would have to be multiplied by a prime smaller than 7.

Our checking <u>proves</u> that 31 is prime.

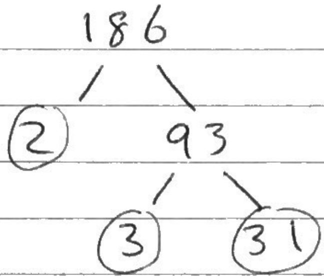
<u>Example</u> 3 Use a factor tree to find the prime factorization of 186.

Solution: we see 186 is even

$$\begin{array}{ccc} & 186 & \\ / & & \backslash \\ 2 & & 93 \end{array}$$

also 3 divides 93

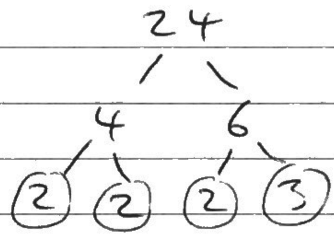
$$\begin{array}{r} 31 \\ 3\overline{)93} \\ -9 \\ \hline 03 \\ 3 \\ \hline 0 \end{array}$$

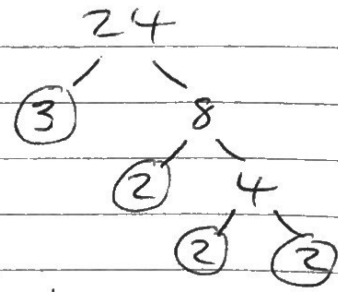


Answer $\boxed{186 = 2 \cdot 3 \cdot 31}$

Example (4) Draw two different factor trees for 24. Do you get different prime factorizations?

Solution:





We get the same prime factorization

$$24 = 2 \cdot 2 \cdot 2 \cdot 3 = \boxed{2^3 \cdot 3}.$$

The greatest common factor of two (or more) numbers is just what it says. Shortened to GCF.

Example (5) Find the GCF of 6 and 8.

Solution: Factors of 6 are 1, 2, 3, 6
Factors of 8 are 1, 2, 4, 8

We see the greatest factor in common is $\boxed{2}$.

Example (6) Find the GCF of 7 and 10.

Solution: factors of 7 are just 1, 7

factors of 10 are 1, 2, 5, 10.
So the GCF of 7 and 10 is $\boxed{1}$. (No factors in common.)

Example (7) Find the GCF of 24 and 60.

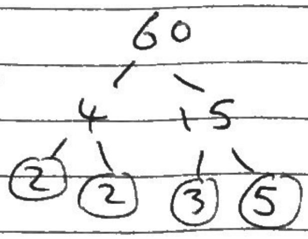
Solution: Listing all the factors is hard work

Factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24
Factors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

$\uparrow$
GCF is $\boxed{12}$.

Better solution: Use the prime factorizations of 24 and 60

$$24 = \underline{\underline{2 \cdot 2}} \cdot 2 \cdot \underline{\underline{3}} \qquad 60 = \underline{\underline{2 \cdot 2 \cdot 3}} \cdot 5$$



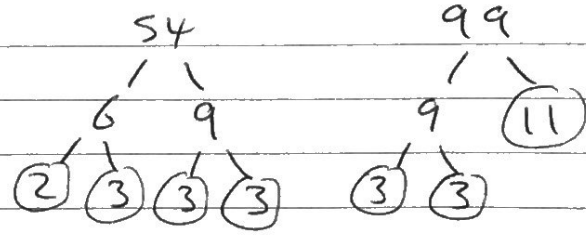
Factors in common $2 \cdot 2 \cdot 3 = \boxed{12}$

Use the GCF to reduce fractions to lowest terms in <u>one step</u>:

$$\frac{24}{60} = \frac{24 \div 12}{60 \div 12} = \frac{2}{5}$$

Example ⑧ Find the GCF of 54 and 99 and use it to simplify $\frac{54}{99}$.

Solution: Factor trees



So $54 = 2 \cdot 3 \cdot 3 \cdot 3$
$99 = 3 \cdot 3 \cdot 11$

The most factors they have in common is $3 \cdot 3$ so $\boxed{GCF = 9}$.

Then $\frac{54}{99} = \frac{54 \div 9}{99 \div 9} = \boxed{\frac{6}{11}}$.

- There are some harder examples on p86, 87.

3.7 Pre-cancelling

Pre-cancelling is a nice shortcut to use when multiplying fractions.

Example ⑨ Multiply $\frac{5}{6} \cdot \frac{9}{10}$ and simplify your answer.

Longer solution: $\frac{5}{6} \cdot \frac{9}{10} = \frac{45}{60}$

we can reduce $\frac{45 \div 5}{60 \div 5} = \frac{9}{12} = \frac{9 \div 3}{12 \div 3} = \boxed{\frac{3}{4}}$

Or in one step (GCF of 45, 60 is 15)

$\frac{45 \div 15}{60 \div 15} = \frac{3}{4}$.

Shorter solution: Look for cancelling before multiplying (so called pre-cancelling).

$$\frac{5}{6}\cdot\frac{9}{10} = \frac{5\cdot 9}{6\cdot 10} = \frac{5\cdot 3\cdot 3}{2\cdot 3\cdot 2\cdot 5}$$

cancel 5s and 3s $= \frac{3}{2\cdot 2} = \boxed{\frac{3}{4}}$

Remember our rule for cancelling

$$\boxed{\frac{a\cdot c}{b\cdot c} = \frac{a}{b}}$$

cancel c

Example (10) Compute $12\cdot\frac{5}{8}\cdot\frac{2}{3}$.

Solution: Write 12 as $\frac{12}{1}$. Factor the numbers and look for matching numbers on the top and bottom:

$$\frac{12}{1}\cdot\frac{5}{8}\cdot\frac{2}{3} = \frac{3\cdot 4}{1}\cdot\frac{5}{4\cdot 2}\cdot\frac{2}{3}$$

Can cancel 2s, 3s and 4s to get $\frac{5}{1} = \boxed{5}$.

Without pre-cancelling you get

$$\frac{12\cdot 5\cdot 2}{8\cdot 3} = \frac{120}{24}$$ and divide to get 5 again.