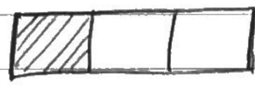
3.4 Multiplying fractions

A good way to picture fractions is using rectangles, as we saw. For example



$\frac{1}{3}$

with the whole rectangle representing 1. If you wanted $\frac{1}{3}$ of a chocolate bar, you would divide it into 3 equal pieces and take one piece.

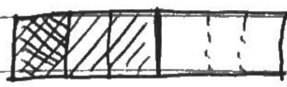
What is $\frac{1}{3}$ of $\frac{1}{2}$? (One third of one half.)

Answer: start with

$\frac{1}{2}$

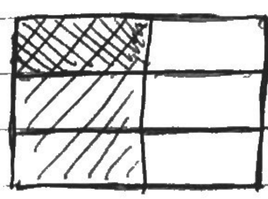
and get $\frac{1}{3}$ of that

? how much of whole rectangle?



$\frac{1}{6}$

So $\frac{1}{3}$ of $\frac{1}{2}$ is $\frac{1}{6}$. Cleerer way to see:



- See p79 in the book for more examples.

multiplication   equals

$\frac{1}{3}$ of $\frac{1}{2}$ is $\frac{1}{6}$

$$\frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$$

"of" always means multiplication.

Other ways to write it $\frac{1}{3} \times \frac{1}{2}$ or $\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)$.
There is a simple rule to multiply two fractions:

$$\boxed{\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}}$$

So you multiply straight across

straight multiply

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

You do **NOT** cross multiply

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ad}{bc}$$

No!
No! No!

Example ① Find one tenth of one fifth.

Answer: $\frac{1}{10} \cdot \frac{1}{5} = \frac{1 \cdot 1}{10 \cdot 5} = \boxed{\frac{1}{50}}$.

(Not equal to 50)

Example ② Compute $\frac{5}{7} \cdot \frac{6}{11}$

Answer: multiply straight across $\boxed{\frac{30}{77}}$.

Example ③ Compute $\frac{5}{7} \cdot 10$

Solution: Here we're multiplying a fraction by a whole number. Remember that $\frac{a}{1} = a$ so

$$\frac{5}{7} \cdot 10 = \frac{5}{7} \cdot \frac{10}{1} = \boxed{\frac{50}{7}}.$$

Example ④ Find $\underbrace{\frac{1}{2} \cdot \frac{3}{4}} \cdot \frac{5}{8}$

Solution: Start with $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$

so $$\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{8} = \frac{3}{8} \cdot \frac{5}{8} = \boxed{\frac{15}{64}}$$

Since all the numerators (tops) get multiplied and all the denominators (bottoms) get multiplied can also do

$$\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{8} = \frac{15}{64}$$

Example ⑤ What is $\left(\frac{4}{3}\right)^3$?

Solution: It means $\frac{4}{3} \cdot \frac{4}{3} \cdot \frac{4}{3}$ which is $\boxed{\frac{64}{27}}$.

- See more examples on p 80 in book.

Some common mistakes

$$\boxed{\frac{3}{8} \cdot \frac{5}{8} = \frac{15}{64} \quad \text{Yes}}$$

$\frac{3}{8} \cdot \frac{5}{8} = \frac{15}{8}$? No

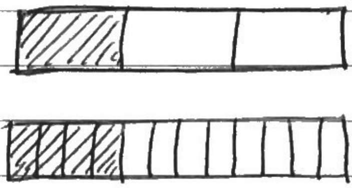
$\frac{3}{8} \cdot \frac{5}{8} = \frac{8}{16}$? No

3.5 Equivalent fractions

Let's compare these two fractions, for example,

$\frac{1}{3}$

and $\frac{4}{12}$



They look like they're really the same. We can show that they are equal using that $\frac{4}{4} = 1$ (remember $\frac{a}{a} = 1$):

$$\frac{4}{12} = \frac{4 \cdot 1}{4 \cdot 3} = \frac{4}{4} \cdot \frac{1}{3} = 1 \cdot \frac{1}{3} = \frac{1}{3}$$

so $\frac{4}{12} = \frac{1}{3}$ and we say $\frac{4}{12}$ and $\frac{1}{3}$ are equivalent fractions. They look different but they're really equal.

Example ⑥ Show that $\frac{6}{10}$ and $\frac{3}{5}$ are equivalent.

Solution: We have $\frac{3}{5} = \frac{3}{5} \cdot 1 = \frac{3}{5} \cdot \frac{2}{2} = \frac{6}{10}$

so $\frac{3}{5} = \frac{6}{10}$ and they are equivalent.

We used $\frac{2}{2} = 1$. Have $\frac{c}{c} = 1$ in general.

Fundamental property of fractions: $\boxed{\frac{a}{b} = \frac{a \cdot c}{b \cdot c}}$

We can use this to make any fraction look more complicated. For example, starting with

$\frac{a}{b} = \frac{1}{4}$ and taking $c = 5$ we get

$$\frac{1}{4} = \frac{1 \cdot 5}{4 \cdot 5} = \frac{5}{20}$$

or take $c = 13$ to get

$$\frac{1}{4} = \frac{1 \cdot 13}{4 \cdot 13} = \frac{13}{52}.$$

c can be any integer. For $c = 100$, $c = 6$ get

$$\frac{1}{4} = \frac{100}{400}, \quad \frac{1}{4} = \frac{6}{24}.$$

We have found four fractions equivalent to $\frac{1}{4}$

$$\frac{1}{4} = \frac{5}{20} = \frac{13}{52} = \frac{100}{400} = \frac{6}{24}$$

and there are infinitely many more.
(Will need this for adding fractions.)

Often we want to make fractions simpler.

Example 7 Simplify $\frac{13}{52}$.

Solution: We're looking for an equivalent fraction with numbers as small as possible.

Know $\frac{13}{52} = \frac{1 \cdot 13}{4 \cdot 13} = \frac{1}{4} \cdot \frac{13}{13} = \frac{1}{4} \cdot 1 = \frac{1}{4}$

So answer is $\boxed{\frac{1}{4}}$.

When we write $\frac{1 \cdot 13}{4 \cdot 13} = \frac{1}{4}$ we say we are <u>cancelling</u> 13.

Nice way to write it: $\frac{13}{52} = \frac{13 \div 13}{52 \div 13} = \frac{1}{4}$

(Not $\frac{13}{52} \div 13 = \frac{1}{4}$).

<u>Example</u> (8) Simplify $\frac{15}{35}$.

Solution: We look for a factor common to top and bottom to cancel. Can use 5

so $\frac{15}{35} = \frac{15 \div 5}{35 \div 5} = \frac{3}{7}$

There are no common factors left in $\frac{3}{7}$ so that's the best we can do.

Answer is $\boxed{\frac{3}{7}}$.

If there is nothing left to cancel then the fraction is in <u>lowest terms</u>.

- More examples p 82, 83.