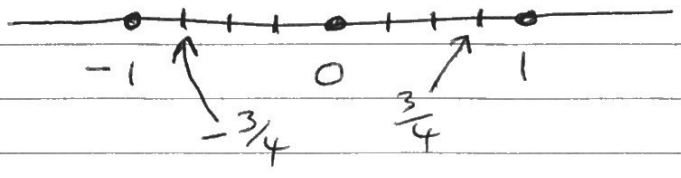
3.12 Signed fractions

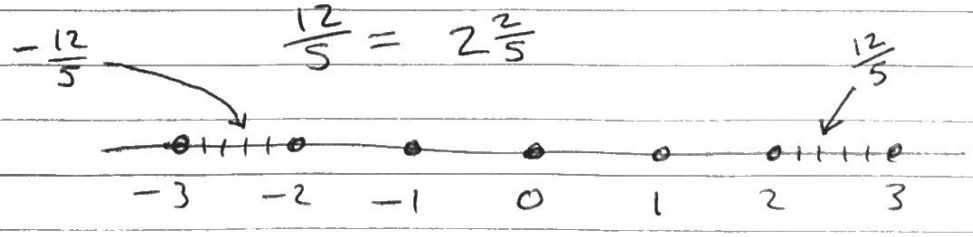
Fractions can be negative as well as positive. To see what is going on we can look on the number line.

Example ① Plot these fractions on the number line: $\frac{3}{4}, -\frac{3}{4}, \frac{0}{3}, \frac{12}{5}, -\frac{12}{5}$

Solution: All the positive proper fractions appear between 0 and 1 on the number line, so $3/4$ is there and so is $\frac{0}{3} = 0$.



Negative fractions are to the left of 0. Converting $\frac{12}{5}$ to a mixed number helps us see where it goes



You can see the left-right symmetry between positive and negative numbers.

Note that $-\frac{12}{5} = -2\frac{2}{5}$ which means $-2-\frac{2}{5}$.

Of course $2\frac{2}{5}$ means $2+\frac{2}{5}$.

How to simplify the sign of a fraction.

Remember that fractions mean division and the rule for dividing two numbers has

same sign ⟶ positive
different sign ⟶ negative

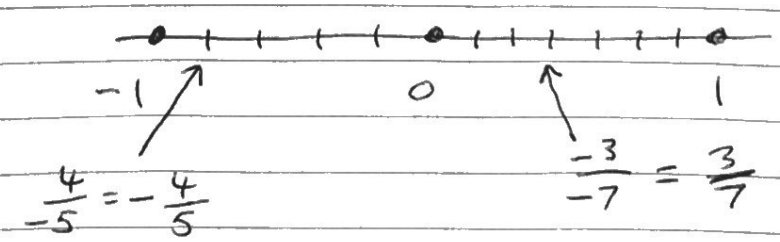
So for example

$$\frac{-3}{-7} = (-3) \div (-7) = +(3 \div 7) = \frac{3}{7}$$

and $$\frac{4}{-5} = 4 \div (-5) = -(4 \div 5) = -\frac{4}{5}.$$

We see that $\frac{-3}{-7}$ is really a positive number

and $\frac{4}{-5}$ is a negative number



Example (2) Simplify the signs of these by deciding if they're positive or negative:

(a) $\frac{-2}{3}$ (b) $\frac{-11}{-3}$ (c) $\frac{4}{-(-9)}$ (d) $\frac{1}{-13}$

Answer: (a) $-\frac{2}{3}$ (b) $\frac{11}{3}$ (c) $\frac{4}{9}$ (d) $-\frac{1}{13}$

negative positive positive negative.

Remember from section 2.3 our

Rule to add two signed numbers

(A) If the numbers have the same sign then add the absolute values and keep the sign.

(B) If the numbers have different signs then subtract the absolute values (larger − smaller) and use the sign of the number with the larger absolute value.

We can use this rule for fractions too.

Example (3) Find $\left(-\frac{3}{4}\right)+\frac{2}{3}$

Solution: We want to use $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$

so we need the LCD which is 12

$$-\frac{3}{4}=-\frac{3\cdot 3}{4\cdot 3}=-\frac{9}{12}=\frac{-9}{12} \qquad a=-9,\ c=12$$

$$\frac{2}{3}=\frac{2\cdot 4}{3\cdot 4}=\frac{8}{12} \qquad b=8,\ c=12$$

Then $\left(-\frac{3}{4}\right)+\frac{2}{3}=\frac{-9}{12}+\frac{8}{12}=\frac{-9+8}{12}$

$-9+8$ use (B) $9-8=1$ } $-9+8=-1$

↑ abs val 9 ↖ abs val 8

$=\frac{-1}{12}=\boxed{-\frac{1}{12}}$.

Example (4) Compute $4\frac{3}{5} - 7\frac{1}{3}$

Solution: As an addition this is $\left(4\frac{3}{5}\right) + \left(-7\frac{1}{3}\right)$

also $4\frac{3}{5} = 4 + \frac{3}{5}$ and $-7\frac{1}{3} = -7 - \frac{1}{3}$.

Altogether have $4 + \frac{3}{5} + (-7) + \left(-\frac{1}{3}\right)$.

Then $4 + (-7) = -3$ ($7-4=3$, use sign of -7)

and $\frac{3}{5} + \left(-\frac{1}{3}\right)$ LCD $= 15$

$= \frac{9}{15} + \left(-\frac{5}{15}\right)$

$= \frac{9}{15} + \frac{-5}{15} = \frac{9-5}{15} = \frac{4}{15}$.

So far we have

$$4\frac{3}{5} - 7\frac{1}{3} = -3 + \frac{4}{15}$$

(Note this is not $-3\frac{4}{15}$ which means $-3 - \frac{4}{15}$.)

To write the answer as a single fraction, or a mixed number we could do this

$$-3 + \frac{4}{15} = -\frac{3}{1} + \frac{4}{15} = \frac{-45}{15} + \frac{4}{15} = \frac{-41}{15}$$

$$= \boxed{-\frac{41}{15}} \quad \text{or} \quad \boxed{-2\frac{11}{15}}.$$

Or, maybe easier, borrow

$$-3 + \frac{4}{15} = -2 - 1 + \frac{4}{15} = -2 - \frac{15}{15} + \frac{4}{15} = -2 - \frac{11}{15} =$$

As you can see in our work it's useful to write

$$-\frac{a}{b} = \frac{-a}{b} \quad \text{or} \quad \frac{-a}{b} = -\frac{a}{b}.$$

For multiplying or dividing two signed fractions or mixed numbers, remember the rule

same sign $\longrightarrow$ positive
different sign $\longrightarrow$ negative.

Also convert the mixed numbers to improper fractions.

- See book examples p111, 112.

3.13 Combined operations

For calculating with fractions, mixed numbers we use the usual order of operations from section 1.7:

(P) Do operations inside grouping symbols first
(E) Exponents and roots next
(MD) Multiplication, division next (left to right)
(AS) Addition, subtraction last (left to right).

Example (5) Find $\frac{2}{5} + \frac{3}{5} \cdot \frac{10}{7}$

Solution: We must do the multiplication before the addition

(M) $\frac{3}{5} \cdot \frac{10}{7} = \frac{3}{5} \cdot \frac{5 \cdot 2}{7} = \frac{6}{7}$

precancel

(A) $\frac{2}{5} + \frac{6}{7} = \frac{2 \cdot 7}{5 \cdot 7} + \frac{6 \cdot 5}{7 \cdot 5}$ (LCD = 35)

$= \frac{14}{35} + \frac{30}{35} = \boxed{\frac{44}{35}}$

Example (6) What is $\left(-\frac{1}{4} - \frac{3}{8}\right) \div \left(-3\frac{1}{2}\right)^2$

Solution: (P) says we must find

$-\frac{1}{4} - \frac{3}{8}$ first $= \left(-\frac{1}{4}\right) + \left(-\frac{3}{8}\right)$

LCD = 8 so get $\frac{-2}{8} + \frac{-3}{8} = \frac{(-2)+(-3)}{8} = \frac{-5}{8}$.

That leaves $\left(-\frac{5}{8}\right) \div \left(-3\frac{1}{2}\right)^2$

(E) exponent next

$\left(-3\frac{1}{2}\right)^2 = \left(-\frac{7}{2}\right)^2 = \left(-\frac{7}{2}\right)\left(-\frac{7}{2}\right)$

same sign so positive $= \frac{7}{2} \cdot \frac{7}{2} = \frac{49}{4}$.

(D) Lastly $\left(-\frac{5}{8}\right) \div \frac{49}{4} = -\left(\frac{5}{8} \cdot \frac{4}{49}\right) = -\frac{5}{2} \cdot \frac{1}{49} = \boxed{-\frac{5}{98}}$.

- More examples p 114, 115, 116.