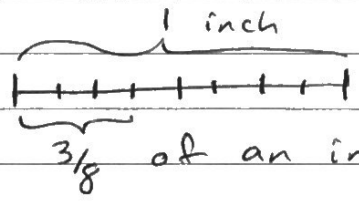
Chapter 3 Fractions and mixed numbers

Fractions look like this example

$$\frac{3}{8}$$

← numerator (top number)
← fraction bar
← denominator (bottom)

You can read it "three over eight" or "three eighths".



$\frac{3}{8}$ means divide 3 into 8 equal pieces

(or divide 1 into 8 equal pieces and take 3 of those pieces).

So $\frac{3}{8} = 3/8 = 3 \div 8$

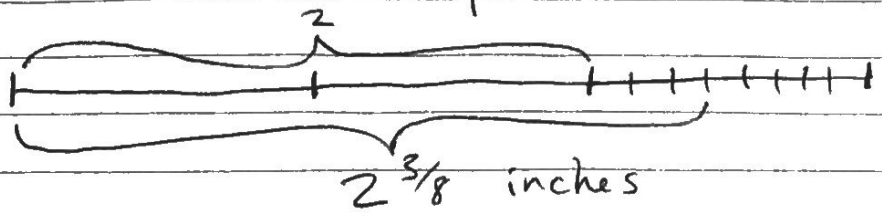
Note that our previous way of studying $3 \div 8$ in terms of quotients and remainders is not very useful:

$$3 \div 8 = 0 \text{ R } 3.$$

A mixed number looks like this

$2\frac{3}{8}$ and means $2 + \frac{3}{8}$.

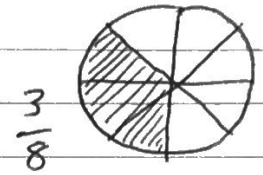
So its a whole number plus a fraction



3.1 Ways to picture fractions

What does it mean to eat $\frac{3}{8}$ of a pizza?

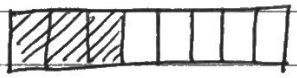
It means you ate 3 of the 8 slices:



$\frac{3}{8}$

Here the whole pizza is $\frac{8}{8} = 1$ and each slice is $\frac{1}{8}$.

As a rectangle:



$\frac{3}{8}$

For these pictures it's important that the pieces have equal size:

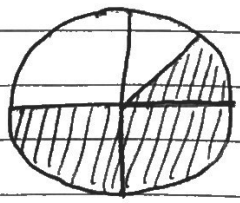
does not represent $\frac{3}{8}$.

Example ① Use a rectangle to represent $\frac{3}{5}$.

Answer:

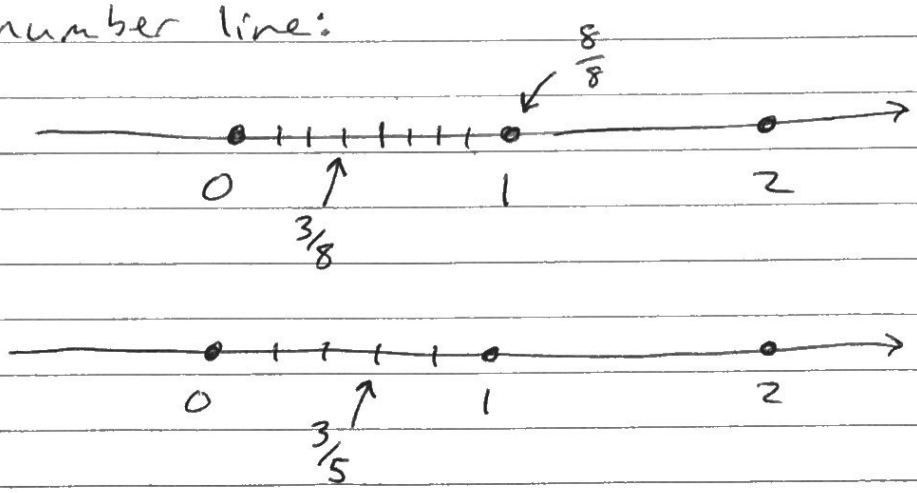
$= \frac{3}{5}$

Example ② What fraction does this picture represent?



Answer: $\frac{5}{8}$ Do you see why?

We can also see where these fractions are on the number line:



See more pictures in the book p 72.

3.2 Proper and improper fractions

A positive fraction $\frac{a}{b}$ is <u>proper</u> if $a<b$.

Otherwise it's <u>improper</u>.

<u>Example:</u> these are all proper $\frac{1}{2}, \frac{7}{8}, \frac{0}{6}, \frac{19}{137}$

and these are all improper $\frac{3}{2}, \frac{6}{6}, \frac{10}{3}, \frac{100}{9}$

We see that proper fractions are always less than 1 and improper fractions represent numbers that are 1 or more.

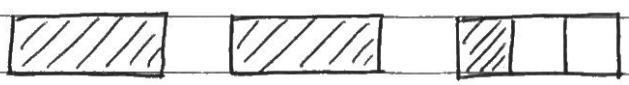
<u>Example (3)</u> Use rectangles to draw the proper fraction $\frac{4}{7}$ and the improper $\frac{7}{4}$.

Answer:  $= \frac{4}{7}$

and  $= \frac{7}{4}$

Each whole rectangle here represents 1.

Example ④ What fraction is this

 ?

Answer: It represents $\frac{7}{3}$

Some simple fraction families:

- $\frac{a}{a}$ is improper, always $= 1$
- $\frac{a}{1}$ is improper, always $= a$
- $\frac{0}{b}$ is proper, always $= 0$
- $\frac{a}{0}$ always undefined

Example: Simplify these fractions

(a) $\frac{4}{1}$, (b) $\frac{9}{9}$ (c) $\frac{0}{19}$ (d) $\frac{215}{215}$ (e) $\frac{3}{0}$ (f) $\frac{3}{1}$

Answers:

(a) 4, (b) 1, (c) 0, (d) 1, (e) undefined, (f) 3.

3.3 Mixed numbers

In example (4) we saw the rectangle picture for $\frac{7}{3}$ had 2 whole rectangles and $\frac{1}{3}$ of a rectangle. So

$$\frac{7}{3} = 2 + \frac{1}{3} = \underbrace{2\frac{1}{3}}_{\text{mixed number notation}}$$

We can write every improper fraction as a mixed number like this.

Example (5) Write $\frac{13}{4}$ as a mixed number.

Solution: Every $\frac{4}{4}$ gives us a whole, so

$$\frac{13}{4} = \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{1}{4} = 1+1+1+\frac{1}{4}$$

$$= 3 + \frac{1}{4} = \boxed{3\frac{1}{4}}.$$

Another way to see this is that

$13 \div 4 = 3$ R 1 (3 ↑ quotient, 1 ↑ remainder) and $\frac{13}{4} = 3\frac{1}{4}$ (3 ↑ quotient; 1 ← rem.; 4 ← divisor (denom.))

Example (6) Write $\frac{82}{5}$ as a mixed number.

Solution:

$$\begin{array}{r} 16 \\ 5\overline{)82} \\ -5\downarrow \\ \hline 32 \\ -30 \\ \hline 2 \end{array}$$

so $82 \div 5 = 16$ R 2 and $\frac{82}{5} = \boxed{16\frac{2}{5}}$.

Example (7) Convert $\frac{84}{7}$ to a mixed number.

Solution: Divide

$$\begin{array}{r} 12 \\ 7\overline{)84} \\ -7\downarrow \\ \hline 14 \\ -14 \\ \hline 0 \end{array}$$

So $\frac{84}{7} = 12\frac{0}{7} = \boxed{12}$.

See chocolate bar example 80 on p. 76.

Other direction — converting mixed numbers back into improper fractions.

Example (8) Convert $7\frac{1}{3}$ to an improper fraction.

Solution: We want to write it as $\frac{\cdot}{3}$

and each whole $1 = \frac{3}{3}$ so $7 = \frac{21}{3}$

and $7\frac{1}{3} = \boxed{\frac{22}{3}}$.

Pattern: $7\left(\frac{1}{3}\right)$ + add $\quad 7\times 3+1 = 22$

$\times$ multiply

$7\frac{1}{3} = \frac{22}{3}$

Example (9) Convert $11\frac{5}{6}$ to an improper fraction.

Solution: $11\times 6+5 = 66+5 = 71$

so $\boxed{\frac{71}{6}}$.