2.6 Dividing signed numbers

Remember the rule to multiply two signed numbers:

- same sign → positive
- different signs → negative

The rule for dividing signed numbers (where there is no remainder) is the same:

The Rule for dividing two signed numbers

Divide their absolute values and
- same sign answer is positive
- different signs answer is negative.

The rules about the signs for multiplication and division are the same because they are opposite (inverse) operations.

Example (1): Find $-30 \div 6$

Answer: $30 \div 6 = 5$ and signs different so $\boxed{-5}$.

Since multiplying by 6 is the opposite of dividing by 6 we see

$$-30 \div 6 = -5 \quad \text{then} \quad -5 \times 6 = -30.$$

Example ② Compute $(-15) \div (-3)$.

Solution: $15 \div 3 = 5$ and signs same so $\boxed{5}$

Example ③ Divide 28 by -7.

Solution: This is $28 \div (-7)$. Different signs so the answer is $\boxed{-4}$.

Other ways to write this example:

$$28/(-7) \quad \text{or} \quad \frac{28}{-7}.$$

Example ④ What is $0 \div (-13)$?

Answer: 0 divided by any number is $\boxed{0}$.

(well any number except 0. Dividing anything by 0 is undefined.)

<u>Remember</u> – a negative number divided by a negative number is always positive.

2.7 Powers of signed numbers

We saw that powers look like 2^3 for example. Here 2 is the base and 3 is the power (or exponent):

$$2^3 = 2 \times 2 \times 2 = 4 \times 2 = 8.$$

When working out powers of signed numbers you have to be very careful when deciding what the base is. Look at these two cases:

A $(-5)^2$ base is -5

B -5^2 base is 5

So A gives $(-5)(-5) = 25$ and B really means $-(5^2) = -(25) = -25$. I hope you see the difference:

$$(-5)^2 = 25$$

$$-5^2 = -25.$$

Example 1 Find $(-3)^3$.

Solution: Base is -3 and
$$(-3)^3 = \underbrace{(-3)(-3)}(-3) = (9)(-3) = \boxed{-27}$$

Example 2 Find $(-2)^6$.

Solution: Base is -2 and
$$(-2)^6 = \underbrace{(-2)(-2)}\underbrace{(-2)(-2)}\underbrace{(-2)(-2)} = (4)(4)(4) = \boxed{64}$$

Note that multiplication is associative so we can multiply in pairs like this.

Example 3 Compute -2^6.

Solution: Now the base is 2. $2^6 = 64$ and

$$-2^6 = -(2^6) = \boxed{-64}.$$

We saw last time that a product is positive if there are an even number of negative factors and negative if there are an odd number of negative factors.

So we have

- even power of a negative is positive
- odd power of a negative is negative

Look at examples 1, 2 again. Why is the answer to example 3 negative?

Example 4 Is $(-12)^{10001}$ a positive or negative number?

Solution: Since the power is odd it must be negative.

Example 5 Find $-(-2)^3$.

Solution: The base is -2 and $(-2)^3 = -8$.

But there is another minus sign to deal with. The minus sign on the left means take the opposite of -8:

$$-(-2)^3 = -\left((-2)^3\right) = -(-8) = \boxed{8}.$$

Example ⑥ Calculate $-(-3^4)$.

Solution: The base is 3 and $3^4 = 81$ so

$$-(-3^4) = \underbrace{-(-81)}_{\text{opposite of } -81} = \boxed{81}$$

Example ⑦ Find $(-1324)^0$.

Solution: Remember that any number to the power 0 equals 1 (well 0^0 undefined) so

$$(-1324)^0 = \boxed{1}.$$

Example ⑧ find $(-1)^{300}$.

Solution: An even power of a negative is positive. Multiply the absolute values to get $1^{300} = 1$. So $(-1)^{300} = \boxed{1}$.

Example ⑨ What is -16^0 ?

Do you see why the answer is $\boxed{-1}$?

2.8 Square roots and signed numbers

We saw that square roots are the opposite (inverse) of squares

$$3 \underset{\text{square root}}{\overset{\text{square}}{\rightleftarrows}} 9 \qquad 3^2 = 3 \times 3 = 9$$

In fact 9 has another square root

$$(-3) \underset{\text{square root}}{\overset{\text{square}}{\rightleftarrows}} 9 \qquad (-3)^2 = (-3)(-3) = 9$$

Every positive number has <u>two</u> square roots, a positive one and a negative one.

Example: 5 and -5 are square roots of 25.

Zero just has one square root, itself.

How about negative numbers?

Example:

$$? \underset{\text{square root}}{\overset{\text{square}}{\rightleftarrows}} -16$$

No number (from the number line) works so we say that negative numbers do not have square roots - or they're undefined:

$\sqrt{-16}$ is undefined.

We use the radical symbol $\sqrt{}$ for square roots and it always means the positive square root:

$$\sqrt{9} = 3$$

Use a minus sign to write the other square root of 9:

$$-\sqrt{9} = -3.$$

Example ① The two square roots of 100 are $\sqrt{100} = 10$ and $-\sqrt{100} = -10$.

Many mistakes are possible!

$$\sqrt{100} = 50 \qquad \text{No.}$$

$$\sqrt{100} = 10 \times 10 \qquad \text{No}$$

$$\sqrt{100} = \sqrt{10} \qquad \text{No}$$

And if you write $\begin{array}{r} 10 \\ 10\overline{)100} \end{array}$ then you're confusing square roots and long division.

Example ② Find $-\sqrt{144}$.

Answer: $\boxed{-12}$

Example ③ Find $\sqrt{-144}$.

Answer: That's undefined.