1.7 Order of operations

We've looked at the operations $+, -, \times, \div$ and powers (sometimes represented with ^). Now we combine them.

A simple example is when we just have $+, -$ as in

- Compute $3+9-2+4$

The rule is to go from left to right and work out the 3 operations in this order

$$\underset{1}{3+}\underset{2}{9-}\underset{3}{2+}4$$

we get

$$= 12 - 2 + 4$$
$$= 10 + 4$$
$$= 14.$$

So answer is $\boxed{14}$.

Note that you <u>don't</u> always add before subtracting.

- Compute $13-1-8+2$

You should get $\boxed{6}$.

The rule for combining multiplication and division is the same - do them in order left to right

- Find $12 \div 6 \times 2$ Solution: $\underbrace{12 \div 6}_{2} \times 2 = \boxed{4}$.

• Evaluate $3 \times 10 \div 3 \times 5$ Check - answer is $\boxed{50}$.

Parentheses and brackets are called grouping symbols and can be used to change the usual order of operations. The rule is to do what is inside the grouping symbols first:

- $10 - 5 + 1 = 6$
- $10 - (5 + 1) = 4$

Here is the full rule for the order of operations

(P) Do operations inside grouping symbols first
(E) Exponents and roots next
(MD) Multiplication, division next (left to right)
(AS) Addition, subtraction last (left to right).

Examples

① Calculate 3×2^3

Solution: No grouping symbols. There is an exponent so do that first $2^3 = 8$. Then the multiplication $3 \times 8 = 24$. Answer $\boxed{24}$.

② Find $4 + 2 \times 6$

Solution: Do the multiplication first $2 \times 6 = 12$ and addition last $4 + 12 = 16$. Ans $\boxed{16}$.

(3) Find $5(3-1)^2 + 4 \div 4^0$

Solution: Using the order of operations rule, check that the correct order is

$$\underset{4\ \text{multiplication}}{5}\underset{1}{(3-1)}^{\overset{2}{2}} \underset{6}{+} 4 \underset{5}{\div} 4^{\overset{3}{0}}$$

and you get $\boxed{24}$.

See more examples in the book, section 1.7. Do them slowly, step by step.

Note that the radical symbol for square roots counts as a grouping symbol. So always work out what is inside the radical before taking the root.

Example $\sqrt{9+16} = \sqrt{25} = \boxed{5}$ Yes

$\sqrt{9+16} = 3+4 = 7$ No

1.8 Averages

A simple application that combines addition, division and parentheses is that of taking an average.
To take the average of a list of numbers you add them all and then divide by the number of numbers in the list.

For example, take the list 3, 2, 6, 4, 5

The sum is $3+2+6+4+5 = 20$ and $20 \div 5 = 4$. So average is $\boxed{4}$. The average is one statistic used to represent a data list with a single number. In a single expression

$$(3+2+6+4+5) \div 5 = 4.$$

Exercise 5 p40. A baseball team had 7 games cancelled in 2010. The number of cancelled games in 2002-2009 were

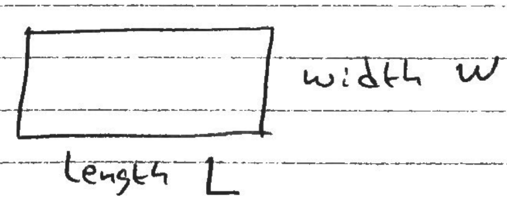
$$5, 6, 2, 10, 9, 4, 6, 5.$$

What was the average number of cancelled games for them for 2002-2010?

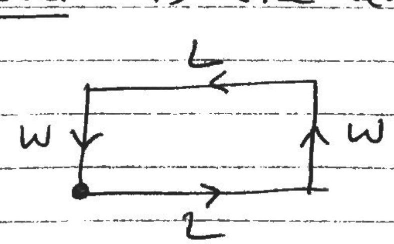
Solution: Average is $(5+6+2+10+9+4+6+5+7) \div 9 = \boxed{6}$.

1.9 Perimeter, Area and the Pythagorean Theorem.

Probably the simplest shape is the rectangle

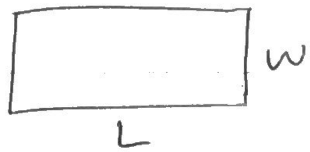


The perimeter is the length around the shape



perimeter $P = 2L + 2W$.

The area measures the space inside using square units



area $A = L \times W$.

Example ① A rectangle has length 10 feet and width 5 feet. Find using the correct units (a) its perimeter and (b) its area.

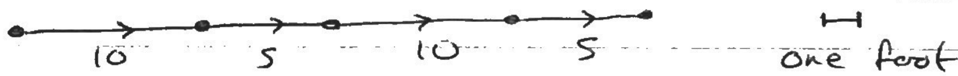
Solution: (a) Perimeter $P = 2L + 2W$

$$= 2 \times 10 + 2 \times 5$$

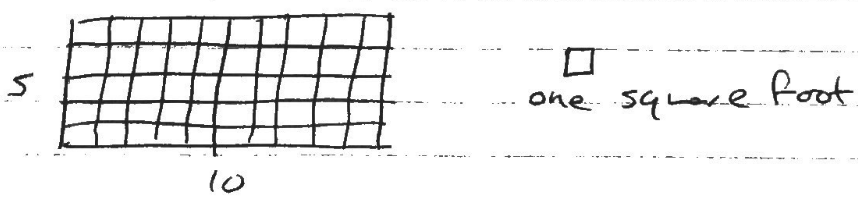
$$= 20 + 10 = 30 \text{ ft}$$

(b) Area $A = L \times W = 10 \times 5 = 50 \text{ ft}^2$.

The correct units for the perimeter here are feet



The correct units for the area here are square feet (notation ft^2)

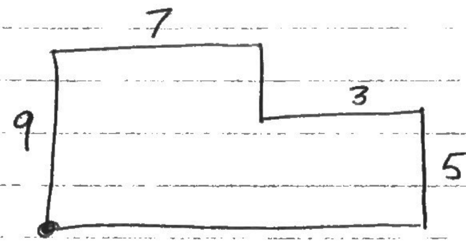


Example ② A rectangle has width 3km and length 7km. Find its area and perimeter with the correct units.

Answer: area $= 21 \text{km}^2$, perimeter $= 20 \text{km}$.

Putting two rectangles together gives an L-shape.

Example (3) Find the perimeter and area of this shape:

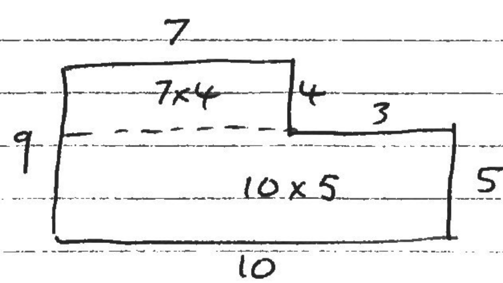


Solution: First find the missing side lengths. The base must be $7+3=10$ units long (units not specified here). Also the short vertical side must be $9-5=4$.

Add all these sides to get the perimeter

$$10+5+3+4+7+9=38.$$

Break the shape into two rectangles to see the area

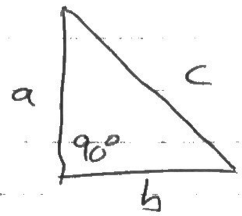


Area top rectangle $= 7 \times 4 = 28$
Area bottom rectangle $= 10 \times 5 = 50$
total 78

Answer $\boxed{\text{perimeter} = 38, \text{ area} = 78}$

The next simplest shape is the triangle.

A right-angled triangle has a 90° angle inside



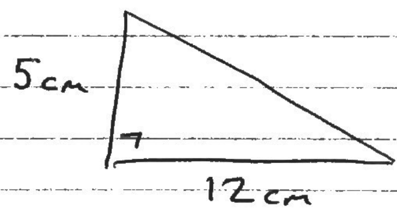
Area $A = (a \times b) \div 2$

The Pythagorean theorem says $a^2 + b^2 = c^2$

and taking square roots of both sides

shows that $c = \sqrt{a^2 + b^2}$. This formula gives the length of the hypotenuse in terms of the two short sides (called legs).

Example ④ Find the area and perimeter of this right-angled triangle



Solution: Here $a = 5$, $b = 12$ and the area

$$A = (a \times b) \div 2 = (5 \times 12) \div 2 = 60 \div 2 = 30\,\text{cm}^2$$

The missing side $c = \sqrt{a^2 + b^2} = \sqrt{5^2 + 12^2} = \sqrt{25 + 144}$

$= \sqrt{169} = 13\,\text{cm}$. So the perimeter $= 5 + 12 + 13 = 30\,\text{cm}$

Answer: $\boxed{\text{Area} = 30\,\text{cm}^2,\ \text{perimeter} = 30\,\text{cm}}$