1.4 Powers of whole numbers

We use powers to write repeated multiplication.

Notation

$$2^3$$

$3 \leftarrow$ power

$2 \leftarrow$ base

$\boxed{2}^{\boxed{3}}$

means $\underbrace{2 \times 2 \times 2}_{\text{3 twos multiplied together}}$ so $2^3 = 8$.

Compare with

$$2 + 3 = 5$$
$$2 \times 3 = 6$$
$$2^3 = 8$$

A power can also be called an exponent.

Any number to the power 1 is just that same number (so we usually dont write a power 1). Any number to the power 0 is defined to be 1 (and 0^0 is undefined).

Examples:

- $13^1 = 13$
- $13^0 = 1$
- $1^0 = 1$
- $0^1 = 0$
- $2^1 = 2$

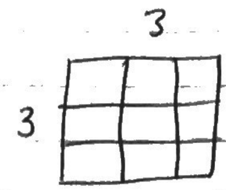
$\boxed{N^1 = N}$

$\boxed{N^0 = 1}$
for $N \neq 0$

Any number to the power 2 just means multiplying the number by itself

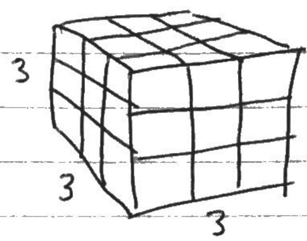
$$N^2 = N \times N.$$

So $3^2 = 9$. We usually say this as "3 squared equals 9" because the area of a square of side length 3 units is 9 square units:

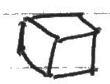


has 9 of these 

Also a third power is often called a cube.



$$\overbrace{3^3}^{3\times3\times3} = 27$$

has 27 of these 

The volume of this cube of side length 3 units is 27 cubic units.

Example: find 15^2

Solution: we need 15×15 so

$$\begin{array}{r} {}^{2} \\ 15 \\ \times\,15 \\ \hline 75 \\ 15 \\ \hline 225 \end{array}$$

Answer $\boxed{15^2 = 225}$

Higher powers don't have names. They can be more work to find by hand

Example: Find 5^4

Solution: $5^4 = \underbrace{5 \times 5}_{25} \times 5 \times 5$

then

$$\begin{array}{r} {\scriptstyle 2} \\ 25 \\ \times\ 5 \\ \hline 125 \end{array} \quad \text{and} \quad \begin{array}{r} {\scriptstyle 1\,2}\ \ \\ 125 \\ \times\ 5 \\ \hline 625 \end{array}$$

Answer $\boxed{5^4 = 625}$

Some numbers are easy to find high powers of. Examples:

- $0^{20} = 0 \times 0 \times \cdots \times 0 = 0$
- $1^{100} = 1 \times 1 \times \cdots \times 1 = 1$
- $10^9 = 1000000000$ ← answer has 9 zeros.

Square roots

Taking a square root is the opposite of squaring. We use the radical notation $\sqrt{\ }$ for square roots (don't confuse with the long division notation).

So $\sqrt{9} = 3$ because $3^2 = 9$

$$\begin{tikzcd} 3 \arrow[r, bend left, "\text{square}"] & 9 \arrow[l, bend left, "\text{square root}"] \end{tikzcd}$$

When you are looking for a square root, for example

$$\sqrt{25} = ?$$

you should ask "which number squared equals 25"? Answer $5^2 = 25$ so

$$\sqrt{25} = 5.$$

Example: find $\sqrt{100}$.

Solution: $\sqrt{100} = 10$ because $10^2 = 100$.

It's a common mistake to confuse square roots with dividing by 2

$$\sqrt{100} = 50 \qquad \text{No.}$$

More examples:

- $\sqrt{0} = 0 \qquad (0^2 = 0)$
- $\sqrt{1} = 1 \qquad (1^2 = 1)$
- $\sqrt{2} =$
- $\sqrt{3} =$
- $\sqrt{4} = 2 \qquad (2^2 = 4)$
- $\sqrt{16} = 4 \qquad (4^2 = 16)$

See section 1.4.3 in the book for more on square roots.

Other common mistakes

- $\sqrt{16} = \sqrt{4} = 2$ No
- $\sqrt{9} = \sqrt{3}$ No
- $\sqrt{9} = 81$ No.

1.5 Division of whole numbers

Division asks how many times the second number fits inside the first.
For example

$$12 \div 4 = 3$$

because 4 fits into 12 3 times. We can also say this as $4 \times 3 = 12$.

How about $13 \div 4$? Well 4 fits into 13 3 times but there is 1 left over:

$$13 \div 4 = 3 \text{ remainder } 1 \qquad (4 \times 3 + 1 = 13).$$

$$= 3 \text{ R } 1$$

Another example: $\underset{\uparrow}{34 \div 5} = \underset{\uparrow}{6}\ \text{R}\ \underset{\uparrow}{4}$

divisor quotient remainder

The remainder can be 0 (which means the divisor divides in evenly with nothing left over). The biggest the remainder can be is one less than the divisor.

Example: $30 \div 7 = 3 \text{ R } 9$

is not correct. We can fit 7 in once more

$$30 \div 7 = 4 \text{ R } 2.$$

We can use <u>Long division</u> for bigger numbers.

Example: find the quotient and remainder for the division:

$$159 \div 6$$

Solution: Set it up like this

$$159 \overline{)\,6} \qquad \text{No}$$

like this!

$$6 \overline{)\,159}$$

6 fits into 1 zero times, but 6 fits into 15 two times. Write the 2 above the 5

$$\begin{array}{r@{\,}l} & 2 \\ 6 & \overline{)\,159} \\ & \underline{-12}\downarrow \\ & 3 \end{array}$$

Bring down the 9 and now we ask how many times 6 fits into 39. Answer is 6 times with 3 left over

$$
\begin{array}{r l}
26 & \leftarrow \text{quotient} \\
6\,\overline{)\,159} & \\
-12 & \\
\hline
39 & \\
-36 & \\
\hline
3 & \leftarrow \text{remainder.}
\end{array}
$$

Answer: $159 \div 6 = 26$ R 3.

See more examples in section 1.5.2.

1.6 Division and 0

We can divide by any number except for one number: zero.

For example $3 \div 0$ causes a problem because 0 fits into 3 infinitely many times. We say

$3 \div 0$ is undefined.

Note that $0 \div 3$ makes sense and $= 0$.

$\boxed{N \div 0 \text{ is undefined}}$ $\boxed{0 \div N = 0}$ for $N \neq 0$