Review of Chapter 6 Towards algebra

The first topic is evaluating expressions which just means replacing variables by numbers. The same steps are needed when using formulas and functions.

<u>Example</u> ① Evaluate $3x^2 - x + 1$ when $x = -4$.

Solution: Replace each x by (-4)

$$3(-4)^2 - (-4) + 1$$

P
E
MD
AS
order of operations

E
$$= 3(16) - (-4) + 1$$

M
$$= 48 - (-4) + 1$$

$48 - (-4)$
$= 48 + 4$

S, A
$$= 52 + 1 = \boxed{53}$$

<u>Example</u> ② Evaluate $a^2 - ab + b$ for $a = \frac{1}{2}$, $b = 3$.

Solution: Get $\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)(3) + (3)$

E $\left(\frac{1}{2}\right)^2 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

to get $\frac{1}{4} - \frac{1}{2} \cdot \frac{3}{1} + 3$

M $\frac{1}{2} \cdot \frac{3}{1} = \frac{3}{2}$

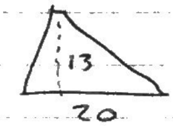
$= \frac{1}{4} - \frac{3}{2} + \frac{3}{1}$

S, A LCD = 4

$= \frac{1}{4} - \frac{6}{4} + \frac{12}{4} = \frac{1 - 6 + 12}{4}$

Answer $\boxed{\frac{7}{4}}$

Example ③ Use the formula $\frac{bh}{2}$ for the area of a triangle with base b and height h to find the area of a triangle with base 20 cm and height 13 cm.



Solution: $\text{area} = \dfrac{(20)(13)}{2}$

$= \dfrac{260}{2} = 130$ Answer $\boxed{130\ \text{cm}^2}$

(Need square units for area.)

Example ④ Find $f(-10)$ for the function $f(x) = x^3 - 9x$

Solution: Replace x by -10:

$$f(-10) = (-10)^3 - 9(-10)$$

E

$$= -1000 - 9(-10)$$

M A

$$= -1000 + 90 = -910$$

Answer $\boxed{-910}$

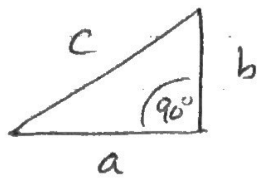
Remember $(-1000) + (90)$ adding numbers with different signs

so subtract absolute values

$$\begin{array}{r} 1000 \\ -\ 90 \\ \hline \cdot\cdot 0 \end{array} \rightarrow \begin{array}{r} {}^{9\,10} \\ \cancel{1}0\cancel{0}\cancel{0} \\ -\ 90 \\ \hline 910 \end{array}$$

and use sign of bigger number.

The Pythagorean Theorem leads to different formulas



$$a^2 + b^2 = c^2$$

Longest side (hypotenuse) $c = \sqrt{a^2 + b^2}$

Also $a^2 = c^2 - b^2$ so

short side $a = \sqrt{c^2 - b^2}$

(short sides a, b called legs

Example ⑤ In a right-angled triangle the hypotenuse is 21 and one leg is 13. Find the length of the remaining side.

Solution: Here can say $c = 21$, $b = 13$

and $a = \sqrt{c^2 - b^2}$

$= \sqrt{(21)^2 - (13)^2}$

$= \sqrt{441 - 169}$

$= \sqrt{272}$

$$\begin{array}{r} 21 \\ \times 21 \\ \hline 21 \\ 42 \\ \hline 441 \end{array}$$

$$\begin{array}{r} 3\,{}^{1}3\,11 \\ \cancel{4}\cancel{4}\cancel{1} \\ -169 \\ \hline 272 \end{array}$$

Answer: The remeining side has length $\boxed{\sqrt{272}}$.

($16^2 = 256$, $17^2 = 289$, so this answer is between 16 and 17.)

Linear equations

The last topic in this chapter is solving linear equations.

A basic example looks like this

$$9x + 14 = 68$$

and to <u>solve</u> it we need to find the number to replace x by that makes the equation true.

Subtract 14 from both sides

$$\begin{aligned} 9x + 14 &= 68 \\ -14 \quad & -14 \\ \hline 9x &= 54 \end{aligned}$$

then divide both sides by 9

$$\frac{9x}{9} = \frac{54}{9}$$

$$1x = 6$$

Answer: $\boxed{\text{the solution is } x = 6}$

Example (7) Solve $3x + 4 = 2$

Solution:

$$\begin{aligned} 3x + 4 &= 2 \\ -4 \quad & -4 \\ \hline 3x &= -2 \end{aligned}$$

$$\frac{3x}{3} = -\frac{2}{3}$$

$$\boxed{x = -\frac{2}{3}}$$

Example ⑧ Solve $9x + 1 = 3x - 4 + 6$

Solution: First note that $-4+6=2$

$$9x + 1 = 3x + 2$$

There are Xs on both sides so subtract one of the linear terms from both sides

$$\begin{array}{rcl} 9x + 1 & = & 3x + 2 \\ -3x & & -3x \\ \hline 6x + 1 & = & 2 \end{array}$$

then continue

$$\begin{array}{rcl} 6x + 1 & = & 2 \\ -1 & & -1 \\ \hline 6x & = & 1 \end{array}$$

and $\frac{6x}{6} = \frac{1}{6} \rightarrow x = \frac{1}{6}$ Answer $\boxed{x = \frac{1}{6}}$

To check our answer, use it in the original equation

$$\underbrace{9\left(\tfrac{1}{6}\right) + 1} \overset{?}{=} 3\left(\tfrac{1}{6}\right) - 4 + 6$$

$$\frac{9}{6} + 1 \qquad \frac{3}{6} - 4 + 6$$

$$\frac{3}{2} + \frac{2}{2} \qquad \frac{1}{2} + 2$$

$$\frac{5}{2} \qquad \frac{5}{2}$$

equal ✓

Example (9) Solve $3(x+2)+2x=-2$

Solution: First use $3(x+2)=3x+6$

to get $3x+6+2x=-2$

then use $3x+2x=(3+2)x=5x$

to get $5x+6=-2$

Continue
$$\begin{array}{rl} 5x+6 &= -2 \\ -6 & \quad -6 \\ \hline \frac{5x}{5} &= \frac{-8}{5} \end{array}$$

Answer $\boxed{x=-\frac{8}{5}}$