

Review for midterm

Chapter 1 Whole numbers

Let's review the basic operations with examples.

Addition: find $469 + 83$

answer 552

$$\begin{array}{r} 469 \\ + 83 \\ \hline 552 \end{array}$$

Subtraction: find $6107 - 4583$

$$\begin{array}{r} 6107 \\ - 4583 \\ \hline 1524 \end{array}$$

Answer 1524

Multiply: compute 394×86

$$\begin{array}{r} 394 \\ \times 86 \\ \hline 2364 \end{array} \rightarrow \begin{array}{r} 394 \\ \times 86 \\ \hline 2364 \end{array}$$

Answer 33884

$$\begin{array}{r} 394 \\ \times 86 \\ \hline 2364 \\ 3152 \\ \hline 33884 \end{array}$$

Division: divide 347 by 9 giving quotient and remainder

$$\begin{array}{r} 9 \overline{) 347} \\ - 27 \\ \hline 7 \end{array} \rightarrow \begin{array}{r} 3 \\ 9 \overline{) 347} \\ - 27 \\ \hline 7 \end{array}$$

Answer 38 R 5

$$\begin{array}{r} 38 \leftarrow \text{quot.} \\ 9 \overline{) 347} \\ - 27 \downarrow \\ \hline 77 \\ - 72 \\ \hline 5 \leftarrow \text{rem.} \end{array}$$

Combining operations

P	()
E	exponents
MD	\times, \div left to right
AS	$+, -$ left to right

examples

$$\bullet 4 + 24 \div 4 = 4 + 6 = \boxed{10}$$

$$\bullet 10 - 2^3 = 10 - 8 = \boxed{2}$$

$$\bullet 9 + 3(2 - \sqrt{16}) \quad \text{do } 2 - \sqrt{16} \text{ first}$$
$$= 2 - 4$$
$$= -2$$

$$= 9 + 3(-2)$$

$$= 9 + (-6)$$

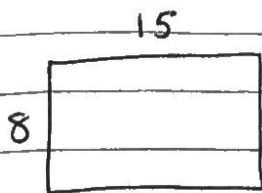
$$= \boxed{3}$$

$$\bullet 10 \times 2 \div 5 \times 2 = 20 \div 5 \times 2$$
$$= 4 \times 2 = \boxed{8}$$

Area, perimeter

We looked at rectangles, L-shapes, triangles

Example. A park is in the shape of a rectangle with length 15 miles and width 8 miles. Find (a) its perimeter (b) its area.



$$(a) \text{ perimeter} = 2 \cdot 15 + 2 \cdot 8$$
$$= 30 + 16 =$$
$$= \boxed{46 \text{ miles}}$$

$$(b) \text{ area} = 15 \cdot 8$$

$$\begin{array}{r} 15 \\ \times 8 \\ \hline 120 \end{array}$$

$$= \boxed{120 \text{ square miles}}$$

Chapter 2 Signed numbers

Absolute value: $|-20| = 20$, $|6| = 6$

Opposite: $-(18) = -18$, $-(-13) = 13$

Multiplying, dividing two signed numbers

Same sign \rightarrow positive
Different signs \rightarrow negative

$$\bullet (-2)(-3) = 6 \qquad 6 \div (-3) = -2$$

$$\bullet 8(-4) = -32 \qquad (-32) \div (-4) = 8$$

Adding two signed numbers

(A) Same sign \rightarrow add abs. vals, keep sign

(B) Different sign \rightarrow subtract abs. vals, use sign of biggest

$$\bullet \begin{array}{ccc} (-12) + (-13) & = & \boxed{-25} \\ 12 \quad 13 & & 12+13=25 \end{array}$$

$$\bullet \begin{array}{ccc} (-12) + 16 & = & \boxed{4} \\ 12 \quad 16 & & 16-12=4 \end{array}$$

$$\bullet \begin{array}{ccc} 18 + (-23) & = & \boxed{-5} \\ 18 \quad 23 & & 23-18=5 \end{array}$$

Subtracting is the same as adding the opposite

$$\bullet (-3) - (6) = (-3) + (-6) = \boxed{-9}$$

3 6 3+6=9

$$\bullet (-4) - (-7) = (-4) + (7) = \boxed{3}$$

4 7 7-4=3

$$\bullet 22 - (-21) = 22 + 21 = \boxed{43}$$

Powers, roots

$$\bullet (-3)^3 = (-3)(-3)(-3) = 9(-3) = \boxed{-27}$$

$$\bullet -(-2)^4 = -((-2)(-2)(-2)(-2))$$
$$= -(4 \cdot 4) = \boxed{-16}$$

$$\bullet -\sqrt{9} = -3$$

$$\bullet \sqrt{-9} \text{ is undefined}$$

$$\bullet (-1)^{123} = -1 \quad (\text{odd powers of negatives are negative, even powers positive.})$$

$$\bullet (-123)^0 = 1$$

$$\bullet 0^4 = 0$$

Chapter 3 Fractions

Representing Fractions

$$\frac{3}{7} = \text{[A rectangle divided into 7 equal vertical strips, with the first 3 strips shaded with diagonal lines.]}$$

$$\frac{2}{5} = \text{[A circle divided into 5 equal sectors, with 2 sectors shaded with diagonal lines.]}$$

$$\frac{8}{5} = \text{[A rectangle divided into 5 equal vertical strips, with 8 strips shaded with diagonal lines. The first 5 strips are in one row, and the remaining 3 strips are in a second row.]}$$

$$= 1\frac{3}{5}$$

Simplifying Fractions

$$\bullet \frac{4}{4} = 1 \quad \bullet \frac{0}{4} = 0 \quad \bullet \frac{8}{4} = 2$$

$$\bullet \frac{2}{4} = \frac{1}{2} \quad \bullet \frac{4}{0} \text{ is undefined} \quad \bullet \frac{4}{1} = 4$$

$$\bullet \frac{10}{40} = \frac{10 \div 10}{40 \div 10} = \frac{1}{4} \quad \frac{a \cdot c}{b \cdot c} = \frac{a}{b}$$

$$\bullet \frac{12}{30} = \frac{12 \div 6}{30 \div 6} = \frac{2}{5}$$

↗
so simplifying this to lowest terms is $\frac{2}{5}$
The GCF of 12 and 30 is 6

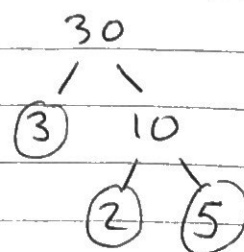
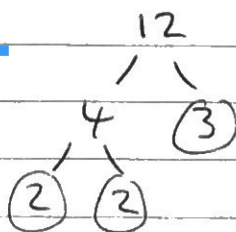
Factors of 12: 1, 2, 3, 4, 6, 12

Factors of 30: 1, 2, 3, 4, 5, 6, 10, 15, 30

Prime Factorizations

$$12 = 2 \cdot 2 \cdot 3$$

$$30 = 2 \cdot 3 \cdot 5$$



GCF = greatest common factor

LCM = least common multiple

LCM of 12 and 30 is 60 because

multiples of 12: 12, 24, 36, 48, 60, 72, ...

multiples of 30: 30, 60, 90, ...

LCD means the least common denominator
(= LCM of all denominators)

The LCD of the fractions $\frac{1}{12}$ and $\frac{7}{30}$
is 60.

compare

Use the LCD to add, subtract fractions

$$\bullet \frac{7}{30} + \frac{1}{12} = \frac{7 \times 2}{30 \times 2} + \frac{1 \times 5}{12 \times 5} = \frac{14}{60} + \frac{5}{60} = \boxed{\frac{19}{60}}$$

$$\bullet \frac{7}{30} - \frac{1}{12} = \frac{7 \times 2}{30 \times 2} - \frac{1 \times 5}{12 \times 5} = \frac{14}{60} - \frac{5}{60} = \frac{9}{60} = \boxed{\frac{3}{20}}$$

To multiply fractions, go straight across
(no LCD needed)

$$\bullet \frac{3}{4} \cdot \frac{5}{7} \begin{matrix} \rightarrow \\ \rightarrow \end{matrix} \frac{3 \cdot 5}{4 \cdot 7} = \boxed{\frac{15}{28}}$$

$$\bullet 15 \cdot \frac{4}{9} = \frac{15}{1} \cdot \frac{4}{9} = \frac{60}{9} = \boxed{\frac{20}{3}}$$

• Multiply $\frac{10}{7} \cdot \frac{14}{9} \cdot \frac{3}{20}$

For this it's a good idea to precancel

$$= \frac{10}{7} \cdot \frac{2 \cdot \cancel{7}}{3 \cdot 3} \cdot \frac{3}{\cancel{10} \cdot 2} = \frac{\cancel{10}}{\cancel{7}} \cdot \frac{2 \cdot \cancel{7}}{3 \cdot \cancel{3}} \cdot \frac{\cancel{3}}{\cancel{10} \cdot 2}$$

look for factors
to cancel on top
and bottom

$$= \boxed{\frac{1}{3}}$$

← when everything
cancels there
is a 1 left

Dividing is the same as multiplying by the
reciprocal

• $\frac{3}{4} \div \frac{9}{5} = \frac{3}{4} \cdot \frac{5}{9} = \frac{15}{36} \div 3 = \boxed{\frac{5}{12}}$

↑ ↑ ↑
keep change flip

Mixed numbers

• Convert $\frac{13}{5}$ to a mixed number: $5 \overline{)13} \begin{array}{r} 2 \\ -10 \\ \hline 3 \end{array}$

$= \boxed{2 \frac{3}{5}}$

• Convert $7 \frac{2}{9}$ to an improper fraction

$$7 \frac{2}{9} = \frac{7 \times 9 + 2}{9} = \frac{63 + 2}{9} = \boxed{\frac{65}{9}}$$

For operations with mixed numbers, convert them to improper fractions first.